INTERMEDIATE MACROECONOMICS

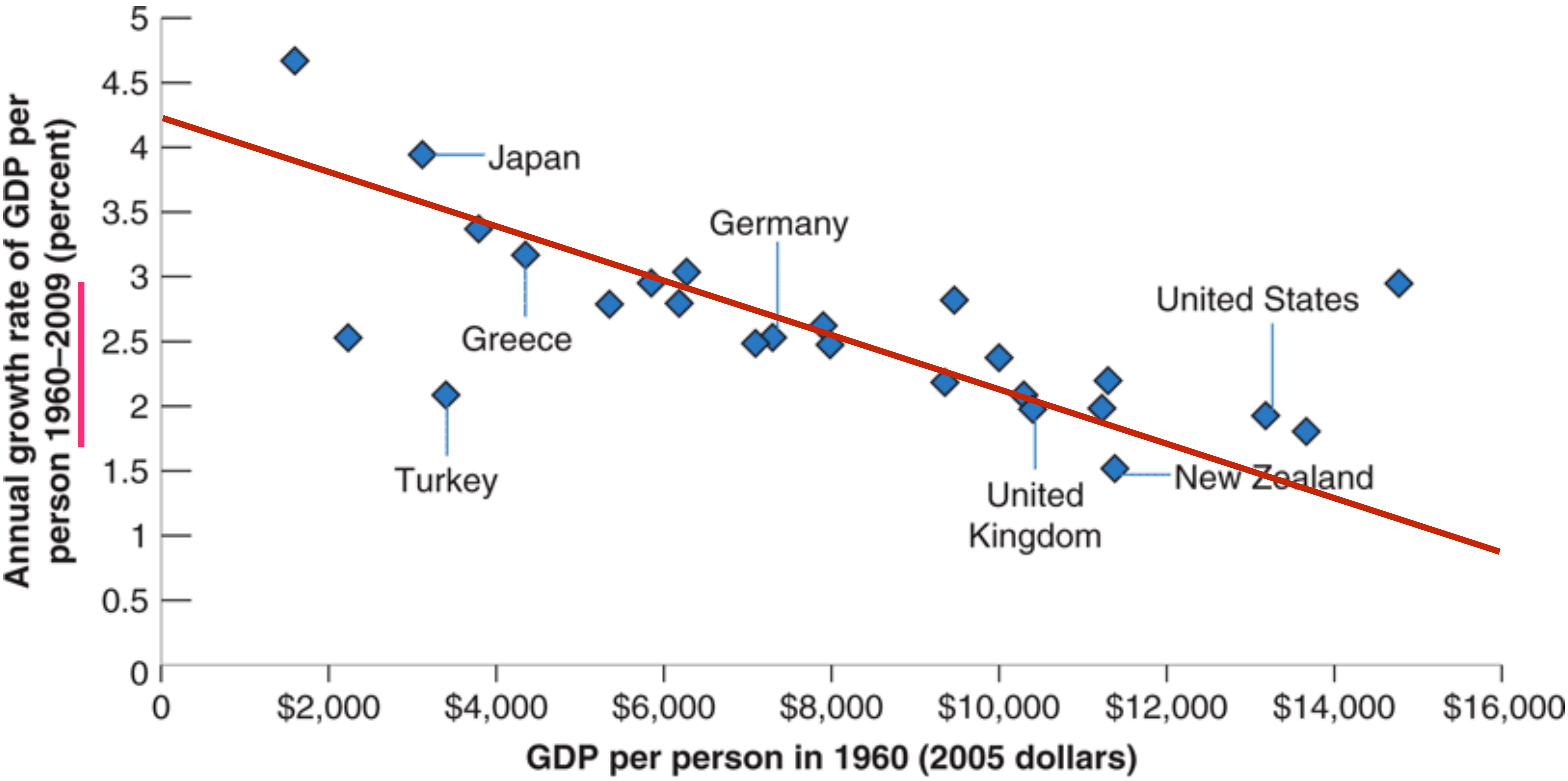
SOLOWIAN MODEL OF GROWTH
25. PRODUCTION & SAVING

Pascal Michaillat
pascalmichaillat.org/c4/

US GROWTH AFTER INDUSTRIAL REVOLUTION

- US real GDP / person: GDP(2014) = 9 × GDP(1890)
  - due to better technology: inventions, production and management techniques, infrastructure, legal and political institutions
- the Malthusian model predicts that technology advancement leads to higher population but same standards of living (Malthusian trap)
  - root cause: the amount of land is fixed
- to explain how technological progress leads to growth in output per worker, we introduce the Solow model of growth
  - production relies on the capital stock, which grows with technology

CONVERGENCE FOR OECD COUNTRIES



Source: See Table 10-1.

PRODUCTION FUNCTION

- aggregate production function: $Y = F(K,N)$
  - Y: output
  - K: capital
  - N: labor
  - the technology level in the economy determines the level of $F(K,N)$
    - in more technologically advanced economies, for a given K and N, $F(K,N)$ is higher

PROPERTIES OF PRODUCTION FUNCTION

- F(K,N) is increasing in K and N
- F(K,N) has constant returns to scale
  - $F(b \times K, b \times N) = b \times F(K,N)$ for any scalar b
- F(K,N) has decreasing returns to capital
  - F(K,N) is concave in K: for a given N, increases in K lead to smaller and smaller increases in F(K,N)
- F(K,N) has decreasing returns to labor
  - F(K,N) is concave in N: for a given K, increases in N lead to smaller and smaller increases in F(K,N)

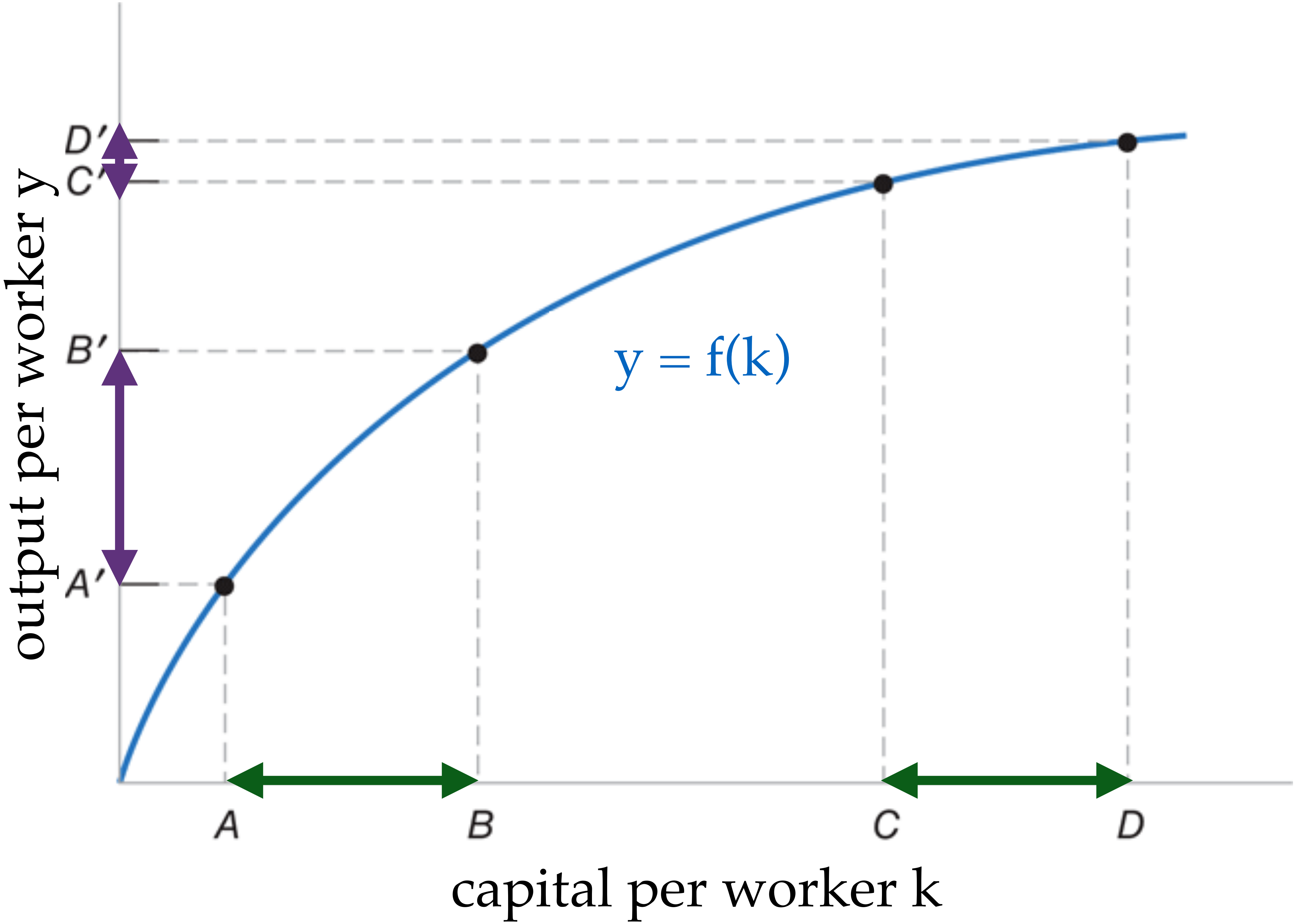
OUTPUT AND CAPITAL PER WORKER

- by constant returns to scale, there is a simple relation between
  - output per worker $y = Y/N$
  - and capital per worker $k = K/N$
- $y = Y/N = F(K,N)/N = F(K/N,N/N) = F(k,1) = f(k)$
  - the function f is defined by $f(x) = F(x,1)$
- f(k) is increasing in k because F(K,N) is increasing in K
- f(k) is concave in k because F(K,N) has decreasing returns to capital

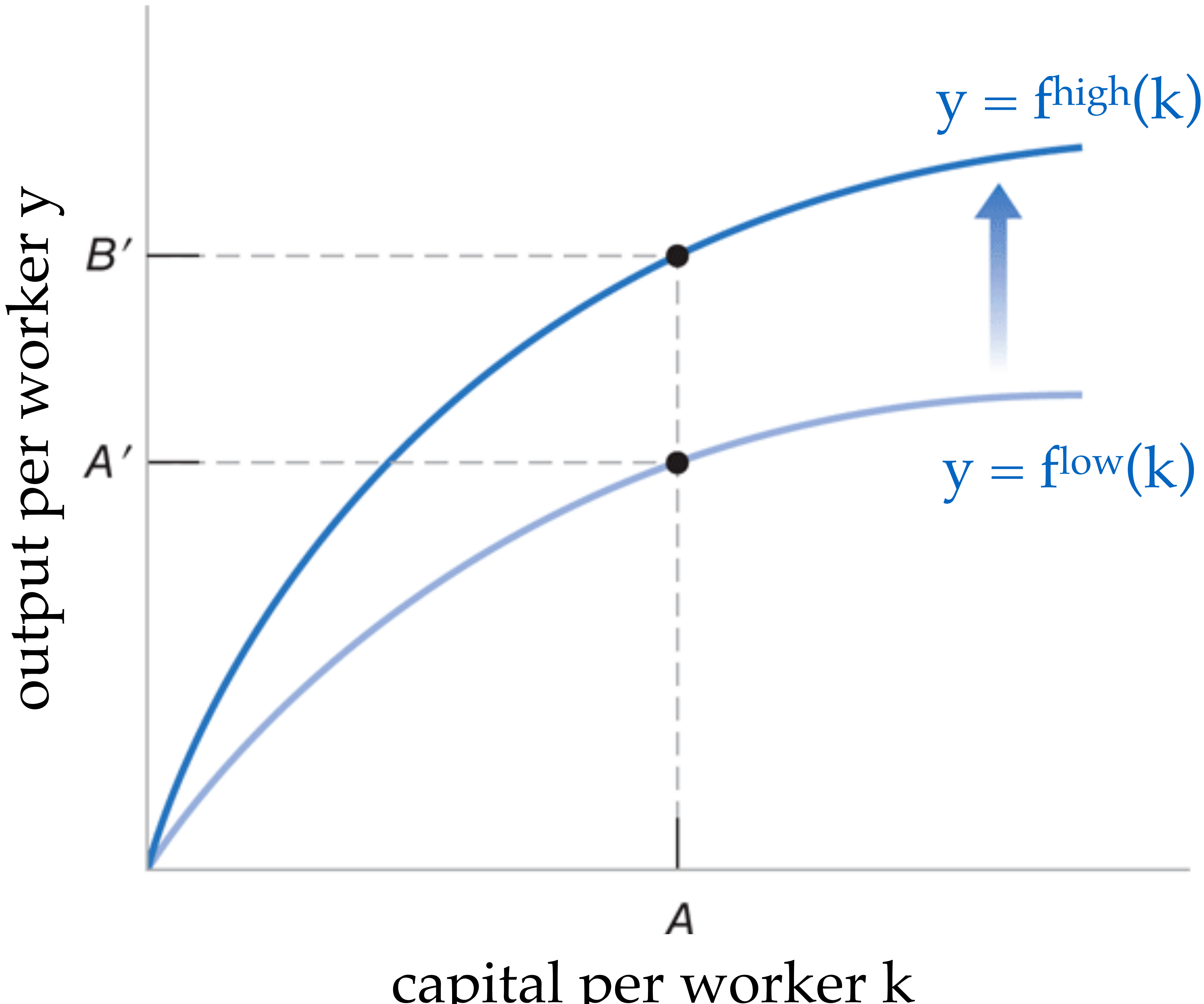
TWO SOURCES OF GROWTH

- output per worker is $y = f(k)$
  - output per worker measures standards of living
- growth of output per worker comes from
  - capital accumulation: a higher k
    - movement along the production function
  - technological progress: a higher $f(.)$
    - shift up of the production function

CAPITAL ACCUMULATION AND DECREASING RETURNS TO CAPITAL



TECHNOLOGY PROGRESS



SAVING BEHAVIOR

- assumption 1: closed economy (no exports or imports)
  - investment = private saving + public saving
- assumption 2: no public saving (government spending = tax revenues)
  - investment = private saving
- assumption 3: private saving S is proportional to income Y
  - $S(t) = s \times Y(t)$
  - $s > 0$ is a parameter measuring the saving rate
- conclusion: output Y and investment I are related by $I(t) = s \times Y(t)$

LAW OF MOTION OF CAPITAL STOCK

- evolution of the capital stock is driven by:
  - investment I: increase in capital stock
  - depreciation $\delta \times K$: decrease in capital stock , because part of the capital stock becomes obsolete
  - $\delta > 0$ is the depreciation rate
- hence: $K(t+1) = K(t) + I(t) - \delta \times K(t)$
- investment comes from saving behavior:
  - $K(t+1) = (1 - \delta) \times K(t) + s \times Y(t)$

SAVING VERSUS DEPRECIATION

- simplifying assumption: employment N is constant
- we divide law of motion of K by N:
  - $K(t+1)/N = (1 - \delta) \times K(t)/N + s \times Y(t)/N$
- hence: $k(t+1) - k(t) = s \times y(t) - \delta \times k(t)$
  - the change in capital per worker = saving per worker minus depreciation per worker
  - if saving per worker > depreciation per worker, capital per worker increases over time

LAW OF MOTION AND STEADY STATE OF CAPITAL PER WORKER

- we use production function to obtain law of motion:
  - $k(t+1) - k(t) = s \times f(k(t)) - \delta \times k(t)$
- law of motion: capital per worker today (k(t)) determines capital per worker tomorrow (k(t+1))
- in steady state, capital per worker is constant:
  - investment per worker = depreciation per worker
  - steady-state capital k^* is such that $s \times f(k^*) = \delta \times k^*$