Samuelson Rule

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Samuelson (1954) rule: $MRS_{gc}\left(g/c\right) = 1$

$[\partial u/\partial g = \partial u/\partial c]$

Samuelson spending: $(g/c)^*$ s.t. $MRS_{gc}((g/c)^*) = 1$

Amount of public spending that satisfies Samuelson rule → optimal public spending in a neoclassical model

First-order approximation of MRS_{gc} (function g/c) around $(g/c)^*$:

$$MRS_{gc} \simeq MRS_{gc}(g/c^*) + \frac{dMRS_{gc}}{d\,g/c} \times \left[g/c - g/c^*\right]$$

(omit all terms of order 2 & above)

$[df = f'(x)\,dx]$

$MRS_{gc}(g/c^*) = 1$

$$\frac{1}{\varepsilon} = -\frac{d\ln MRS_{gc}}{d\ln g/c} = -\frac{g/c}{MRS_{gc}} \times \frac{dMRS_{gc}}{d\,g/c}$$

derivative evaluated at g/c^*:

$$\frac{dMRS_{gc}}{d\,g/c} = \frac{-1}{\varepsilon} \cdot \frac{MRS_{gc}(g/c^*)}{g/c^*} = \frac{-1}{\varepsilon} \cdot \frac{1}{g/c^*}$$

$$\Rightarrow MRS_{gc} = 1 - \frac{1}{\varepsilon} \cdot \frac{g/c - g/c^*}{g/c^*}$$

$$1 - MRS_{gc} = \frac{1}{\varepsilon} \cdot \frac{g/c - g/c^*}{g/c^*}$$

elasticity of substitution b/w public & private goods

departure from Samuelson spending

= stimulus spending