# Efficient Unemployment and Unemployment Gap

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CBO : natural rate of unemployment

Taking a trend of unemployment rate + adjustments

Premise on average, labor market is efficient

Problem No guarantee that labor market is efficient on average → in matching model, no reason to believe that labor market is efficient.

Phillips-curve approach accelerationist Phillips curve

(Friedman) : target unemployment rate such that inflation remains constant

Problem
- care about other things than keeping inflation constant
- complete disconnect between inflation & unemployment

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Efficient unemployment rate in matching model

Efficient maximizes social welfare.

Social welfare: sum of welfare of all individuals
utility

Assumption Cobb-Douglas matching function

## Assumptions to simplify social welfare

- Linear utility function over consumption (risk neutral)

→ all individuals value consumption the same

→ can compute aggregate utility from consumption by aggregating consumption = output.

- Disutility from work = disutility from searching for a job → value of time is the same for employed & unemployed workers

→ value of time is not relevant for welfare.

⇒ social welfare is determined solely by aggregate consumption = aggregate output.

**Definition** efficient unemployment rate is the unemployment rate that maximizes output.

Social planner. benevolent government that can allocate workers between unemployment, producing, & recruiting in order to maximize welfare = output

Social planner is subject to matching function, production function, recruiting process, etc.

Social planner picks <u>\# vacancies</u> to maximize output $\Leftrightarrow$ picking <u>tightness</u> to maximize output.

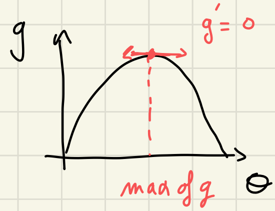
<u>Efficient unemployment rate is unemployment rate chosen by social planner</u>

## Solution to social planner's problem

$$\max_{V} Y \quad \Big| \quad \cdot\ Y = a\, \underline{N^{\alpha}} \qquad \cdot\ \text{picking } V \Leftrightarrow \text{picking } \theta$$

$$\Leftrightarrow \max_{\theta} N(\theta) \longrightarrow \begin{cases} \text{workers} = \text{producers} + \text{recruiters} \\ L = [1+\underset{\substack{\uparrow \\ \text{recruiter/producer}}}{\tau(\theta)}]\, N \end{cases}$$

$$\Leftrightarrow \max_{\theta} \left.\overbrace{\frac{L^{s}(\theta)}{1+\tau(\theta)}}\right\} g(\theta)$$




<u>Necessary condition for maximum</u> $\dfrac{dg}{d\theta} = 0$

$$\Leftrightarrow \frac{\theta}{g} \cdot \frac{dg}{d\theta} = 0 \Leftrightarrow \frac{d \ln g}{d \ln \theta} = 0 \Leftrightarrow \boxed{\varepsilon^{g}_{\theta} = 0}$$

$$\varepsilon_\theta^g = \varepsilon_\theta^{L^S} - \varepsilon_\theta^{1+\tau}$$

$(1-\eta)\, u(\theta)$ $\quad$ $\eta \cdot \tau(\theta)$

$$L^S = \frac{f(\theta)}{s+f(\theta)} H$$

$$\varepsilon_\theta^{L^S} = \varepsilon_\theta^{f/s+f}$$

$$\varepsilon_\theta^g = (1-\eta)\, u(\theta) - \eta\, \tau(\theta)$$

Solution to social planner's problem $\theta$ that maximizes social welfare = output = # of producers is given by

$$(1-\eta)\, u(\theta) = \eta\, \tau(\theta)$$

$$\boxed{\frac{u(\theta)}{\tau(\theta)} = \frac{\eta}{1-\eta}}$$

→ efficient labor market tightness $\theta^*$

→ efficient unemployment rate $u^* = u(\theta^*) = \dfrac{s}{s+f(\theta^*)}$

→ efficient level of output $= a\, N(\theta^*)^\alpha$

$$= a \left[\frac{L^S(\theta^*)}{1+\tau(\theta^*)}\right]^\alpha$$

Application to the US labor market

Research on matching function (Petrongolo & Pissarides, 2001).

$\eta = 0.5 \quad \Rightarrow \quad \eta/1-\eta = 1$

In practice labor market is efficient when $\boxed{\frac{u}{\tau} = 1}$

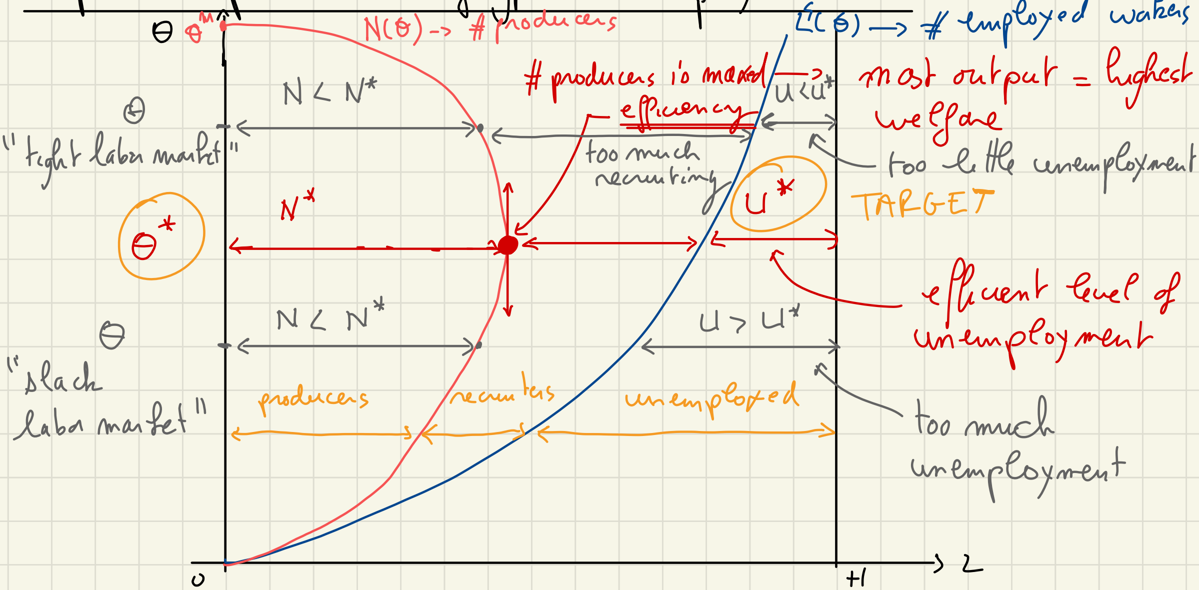
Rule of thumb.

\# unemployed workers = # recruiters

idle

non-productive

Graphical representation of efficient unemployment rate




In real world wage function may not guarantee that $\theta = \theta^*$ → firms may not have the incentive to post a # of vacancies such that $\theta = \theta^*$ and $U = u^*$ and so on → government intervention may be needed to bring labor market closer to efficiency.

## Efficient unemployment rate in Beveridgean models

Assumption: Model admits a Beveridge curve

$$\boxed{v = v(u)}$$

where the function $v(u)$ is decreasing, convex

Examples:
- Matching model
- Mismatch model
- Stock-flow matching model

Advantage: Many countries have a Beveridge curve → method is applicable

Two key parameters:
- recruiting cost $r$ workers/vacancy
- social value of unemployment time / employment time $z$

$z > 0$ or $z < 0$.

Social welfare: $SW = \underbrace{(H - U) - V \times r}_{\substack{\#\text{ employed workers } - \#\text{ recruiters} \\ = \text{ producers} \\ = \text{ output}}} + \underbrace{z\, U}_{\text{"output from unemployment"}}$

Social welfare / capita = $\frac{SW}{H} = (1-u) - v \times r + z \cdot u$

↳ $\boxed{sw(u) = (1-u) + z\,u - v(u) \times r}$

Efficient unemployment rate $u^*$ maximizes

$$sw(u) = \underbrace{\underbrace{1-(1-z)\,u}_{\text{linear}} - \underbrace{v(u) \times r}_{\text{convex}}}_{\text{concave function}}$$

Necessary condition for a maximum of the social welfare function is : $\frac{dsw}{du} = 0$ (first-order condition)

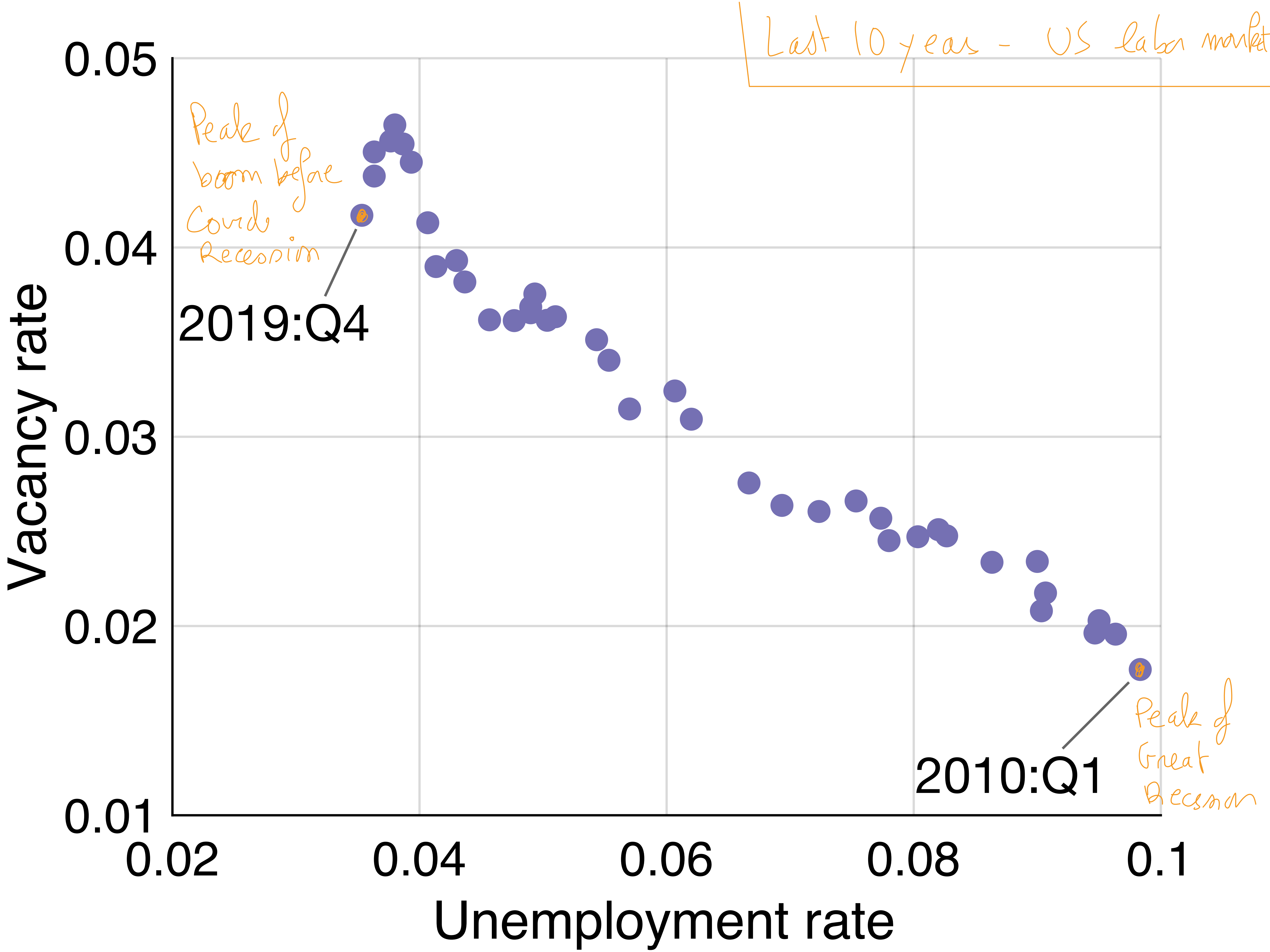
Since $sw(u)$ is concave necessary condition is also sufficient → any $u$ such that $dsw/du = 0$ is a maximum (maximum will be unique)

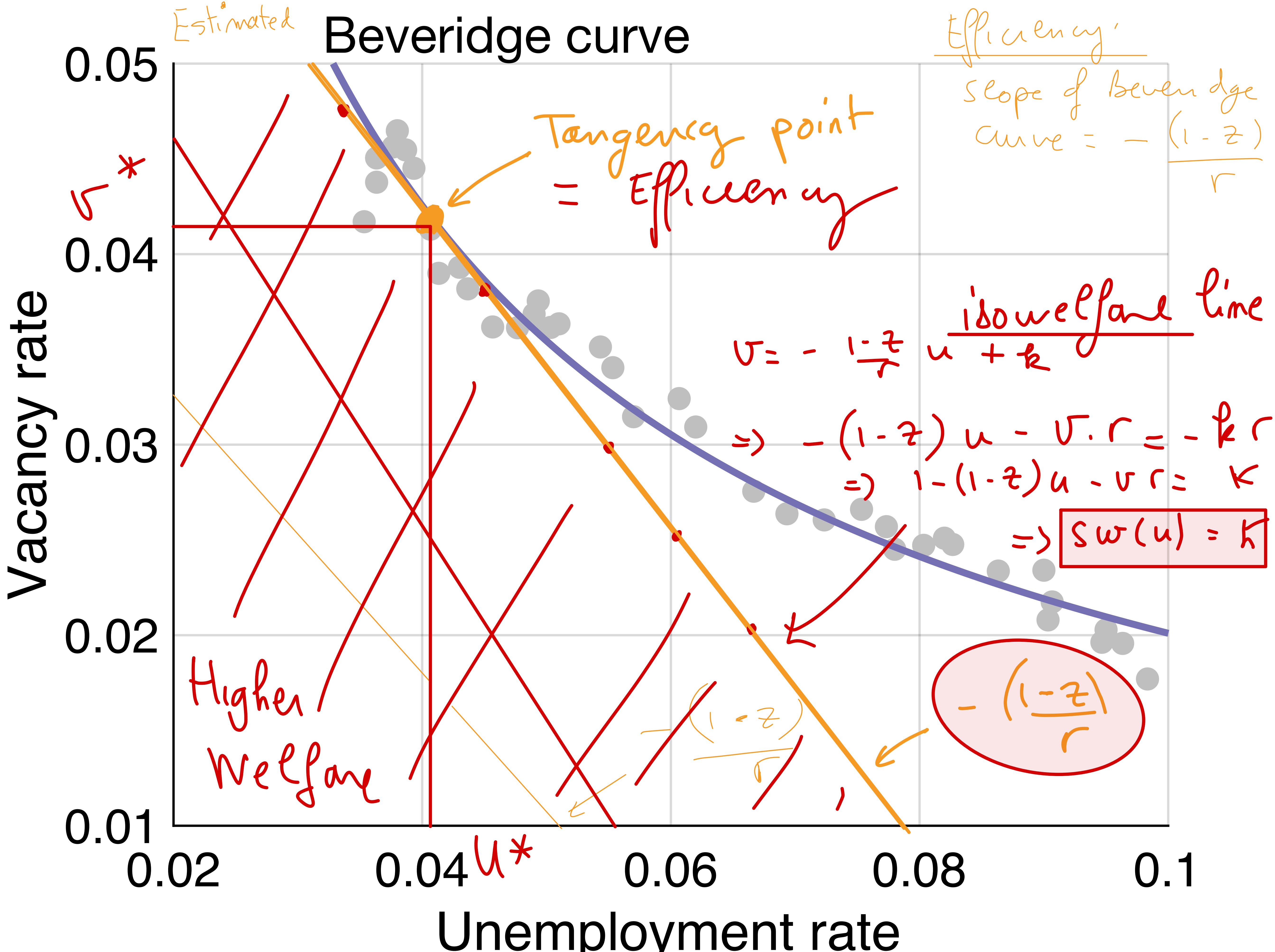
Efficient unemployment rate satisfies

$$\frac{dsw}{du} = -(1-z) - v'(u) \times r = 0$$

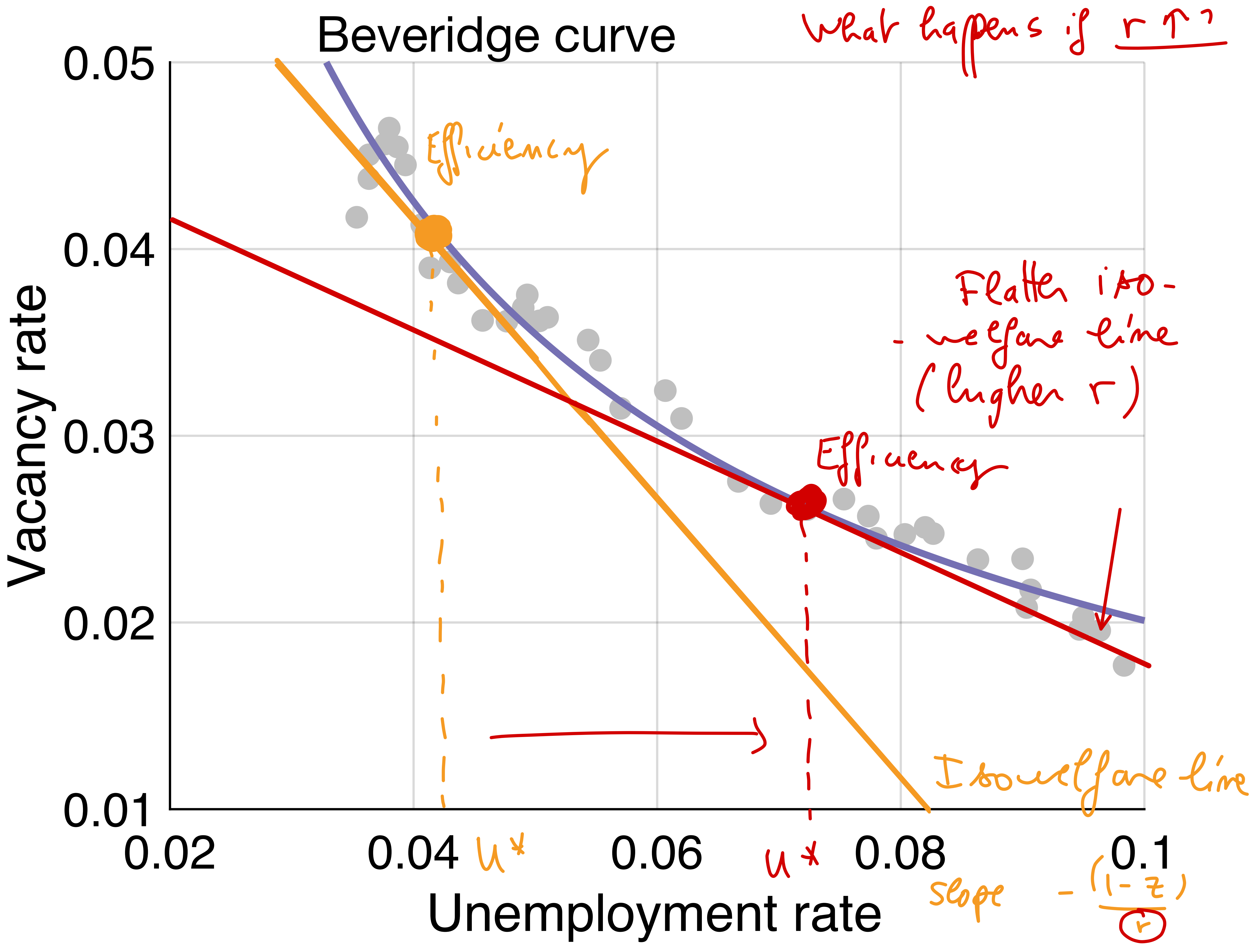
$$\Rightarrow \boxed{v'(u^*) = -\frac{1-z}{r}}$$

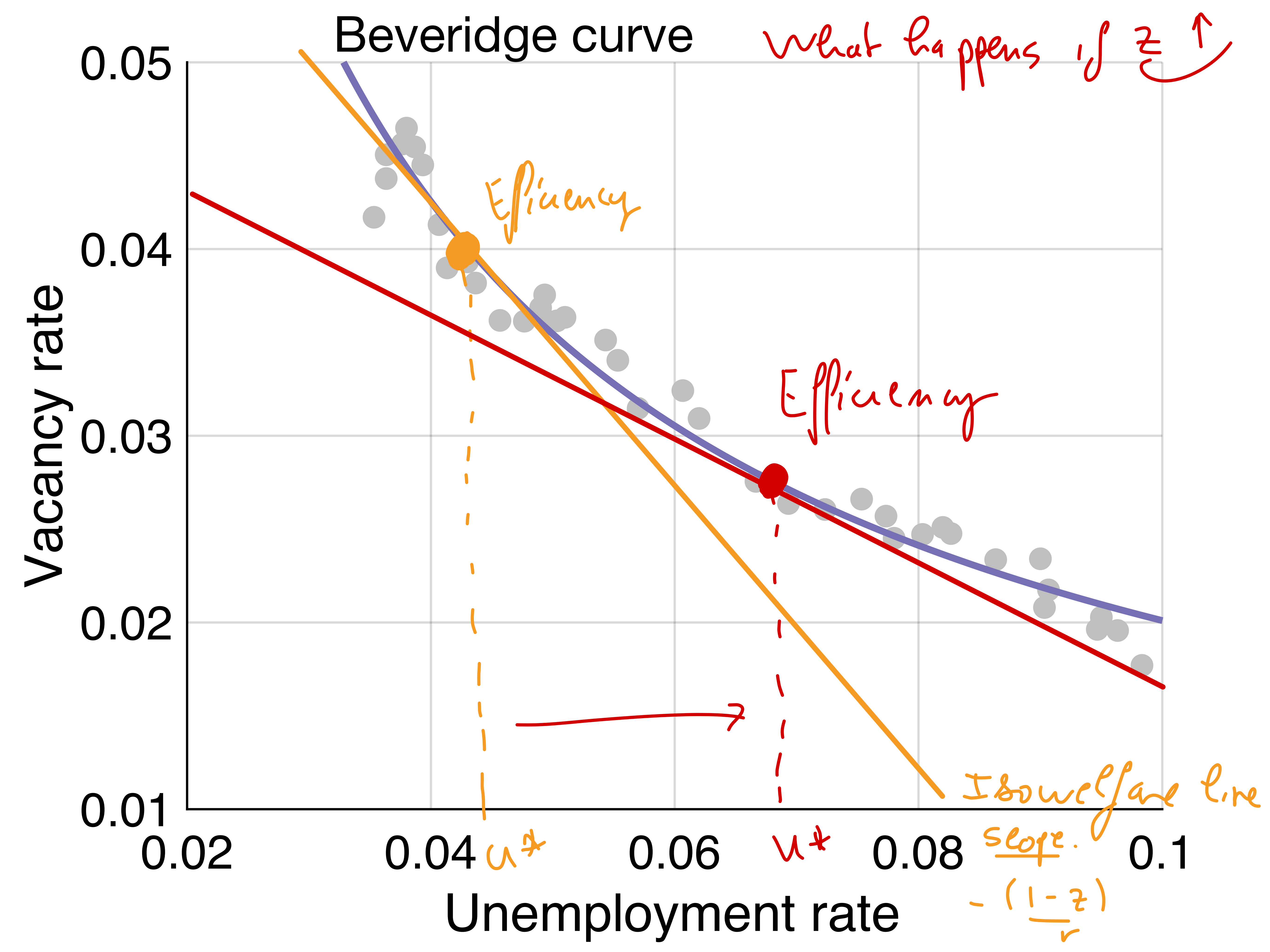
- Slope of Beveridge curve $= -(1-z)/r$
- $v(u)$ is convex → $v'(u)$ is strictly increasing → efficient unemployment rate is unique
- $r \uparrow \Rightarrow u^* \uparrow$ | • $z \uparrow \Rightarrow u^* \uparrow$


Last 10 years - US labor market
Peak of boom before Covid Recession
2019:Q4
2010:Q1
Peak of Great Recession
Vacancy rate
Unemployment rate
0.05
0.04
0.03
0.02
0.01
0.02
0.04
0.06
0.08
0.1

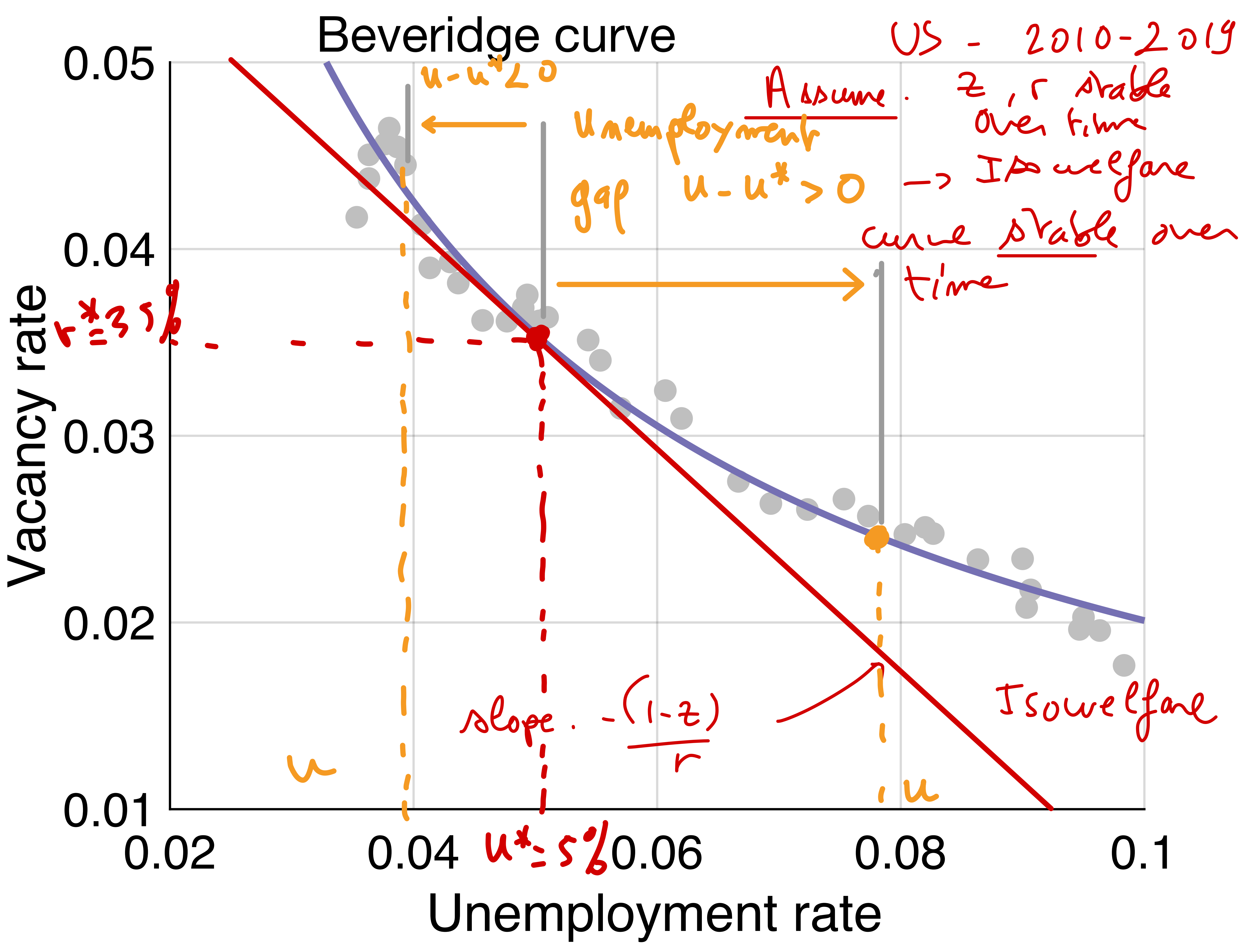


Beveridge curve
Estimated
Vacancy rate
Unemployment rate
0.05
0.04
0.03
0.02
0.01
0.02
0.04
0.06
0.08
0.1
v*
u*
Tangency point
= Efficiency
Efficiency:
Slope of Beveridge curve = −(1−z)/r
isowelfare line
v = −(1−z)/r u + k
⇒ −(1−z) u − v·r = −k r
⇒ 1−(1−z) u − v r = k
⇒ SW(u) = k
−(1−z)/r
Higher Welfare


What happens if r ↑ ?
Beveridge curve
Vacancy rate
0.05
0.04
0.03
0.02
0.01
0.02
0.04
0.06
0.08
0.1
Unemployment rate
Efficiency
Flatter iso-welfare line (higher r)
Efficiency
Isowelfare line
u*
u*
slope $-\frac{(1-z)}{r}$



Beveridge curve
What happens if z ↑
Efficiency
Efficiency
Isowelfare line
slope.
$-\frac{(1-z)}{r}$
u*
u*
Vacancy rate
Unemployment rate
0.05
0.04
0.03
0.02
0.01
0.02
0.04
0.06
0.08
0.1



Beveridge curve
US - 2010-2019
Assume: z, r stable over time
→ Isowelfare
curve stable over time
Unemployment gap u - u* > 0
u - u* < 0
v* = 3.5%
u* = 5%
slope: -(1-z)/r
Isowelfare
u
u
0.05
0.04
0.03
0.02
0.01
Vacancy rate
0.02
0.04
0.06
0.08
0.1
Unemployment rate

# Formula for the efficient labor market tightness

Condition for labor market efficiency: $\boxed{v'(u^*) = -\frac{1-z}{r}}$

Tightness $\theta = v/u$

Beveridge elasticity
(normalized to be positive) $\boxed{\varepsilon = -\frac{d\ln v}{d\ln u}}$

$$\varepsilon = -\frac{u}{v}\cdot\frac{dv}{du} = -\frac{v'(u)}{\theta} \leftarrow$$

Efficiency condition $\theta\,\underbrace{\frac{-v'(u)}{\theta}}_{\varepsilon} = \frac{1-z}{r}$

Efficiency condition $\boxed{\theta^* = \frac{1-z}{\varepsilon\, r}}$

3 key factors

- $z$ value of unemployment

  $z \uparrow \Rightarrow \theta^* \downarrow, \; u^* \uparrow$

- $r$ recruiting cost

  $r \uparrow \Rightarrow \theta^* \downarrow, \; u^* \uparrow$

- $\varepsilon$ elasticity of Beveridge curve

  $\varepsilon \uparrow \Rightarrow \theta^* \downarrow, \; u^* \uparrow$

## Application to US labor market

- 2.5% of labor costs devoted to recruiting (US 1997)
  ↳ 2.5% of workers are recruiters

$$\boxed{\tau \simeq 0.7}$$

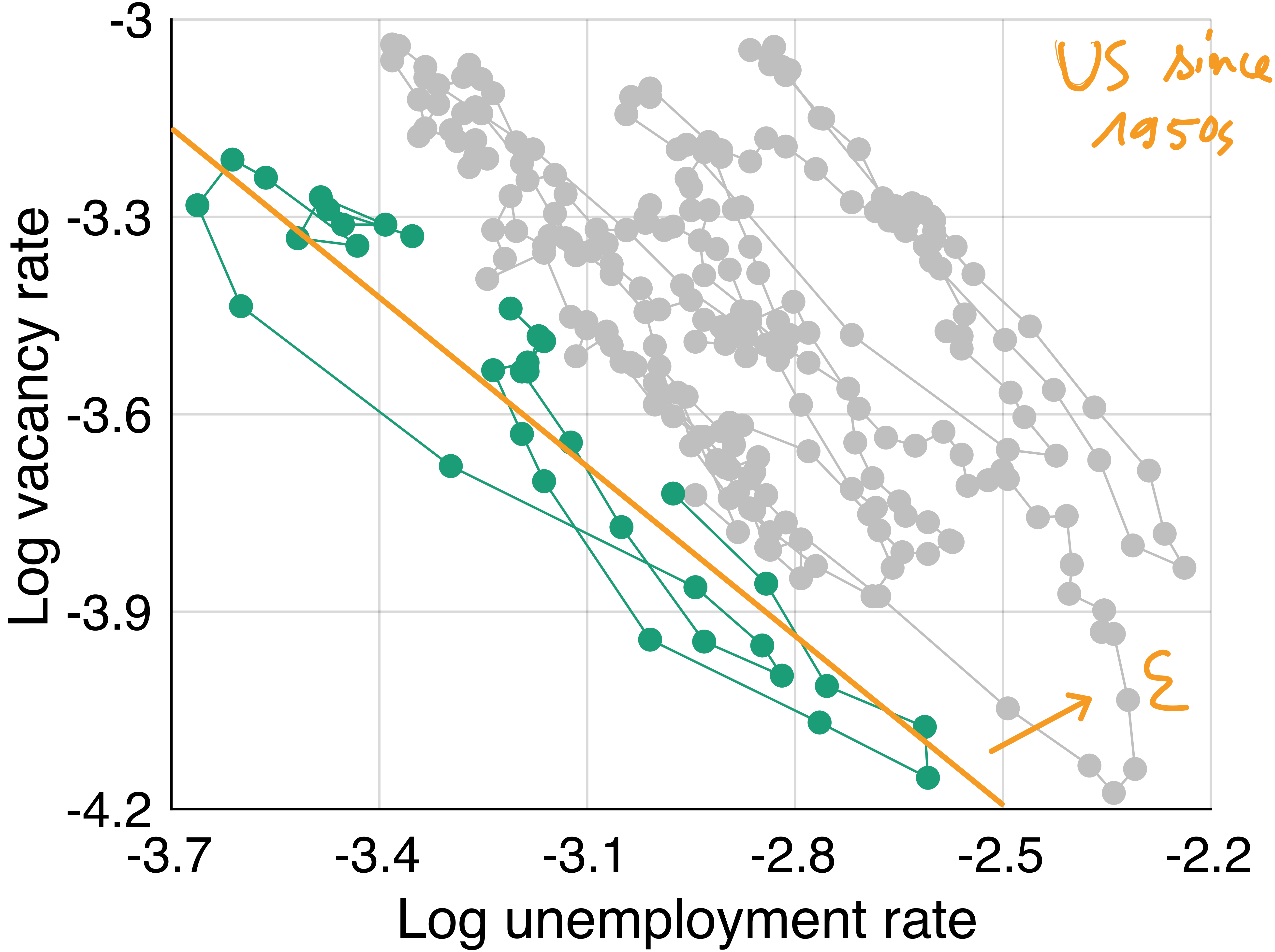
- 13% - 35% of lost earnings ($\simeq$ labor productivity) replaced by leisure & home production

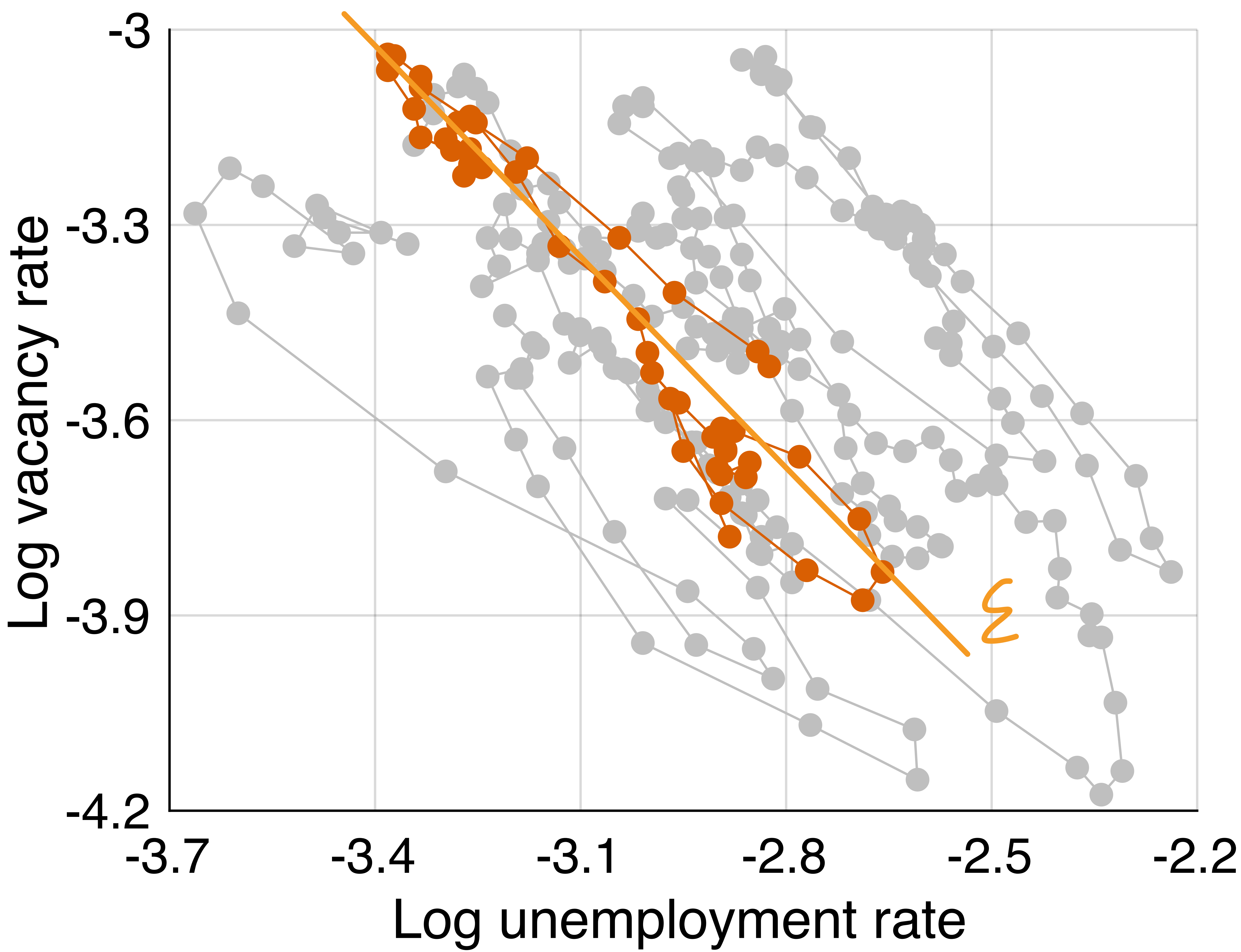
$13\% \leq z \leq 35\% \to \boxed{z \simeq 1/4}$

- $\varepsilon = \dfrac{d \ln v}{d \ln u}$

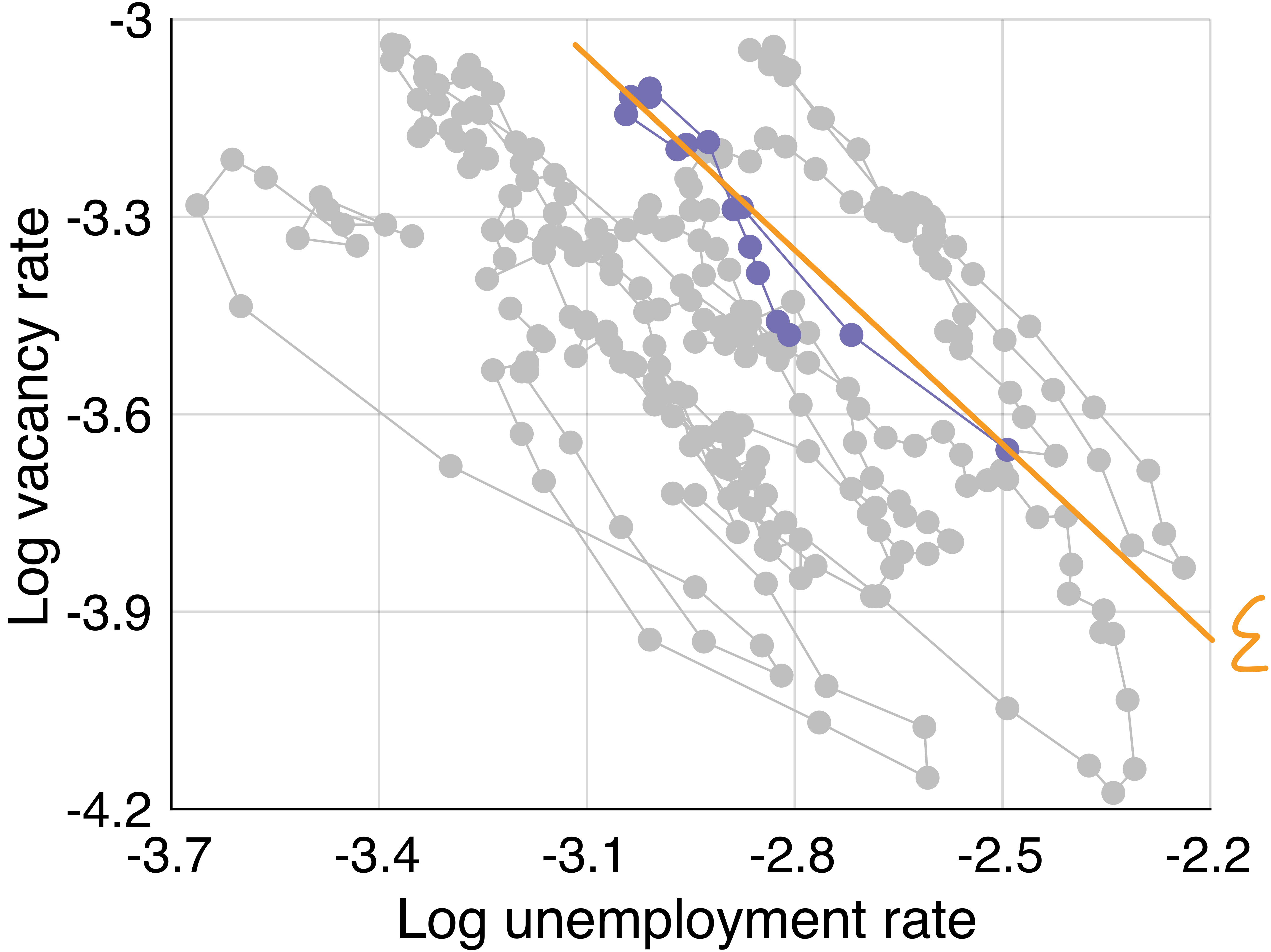
Slope of curve : $\ln v$ versus $\ln u$.

Coefficient in regression.

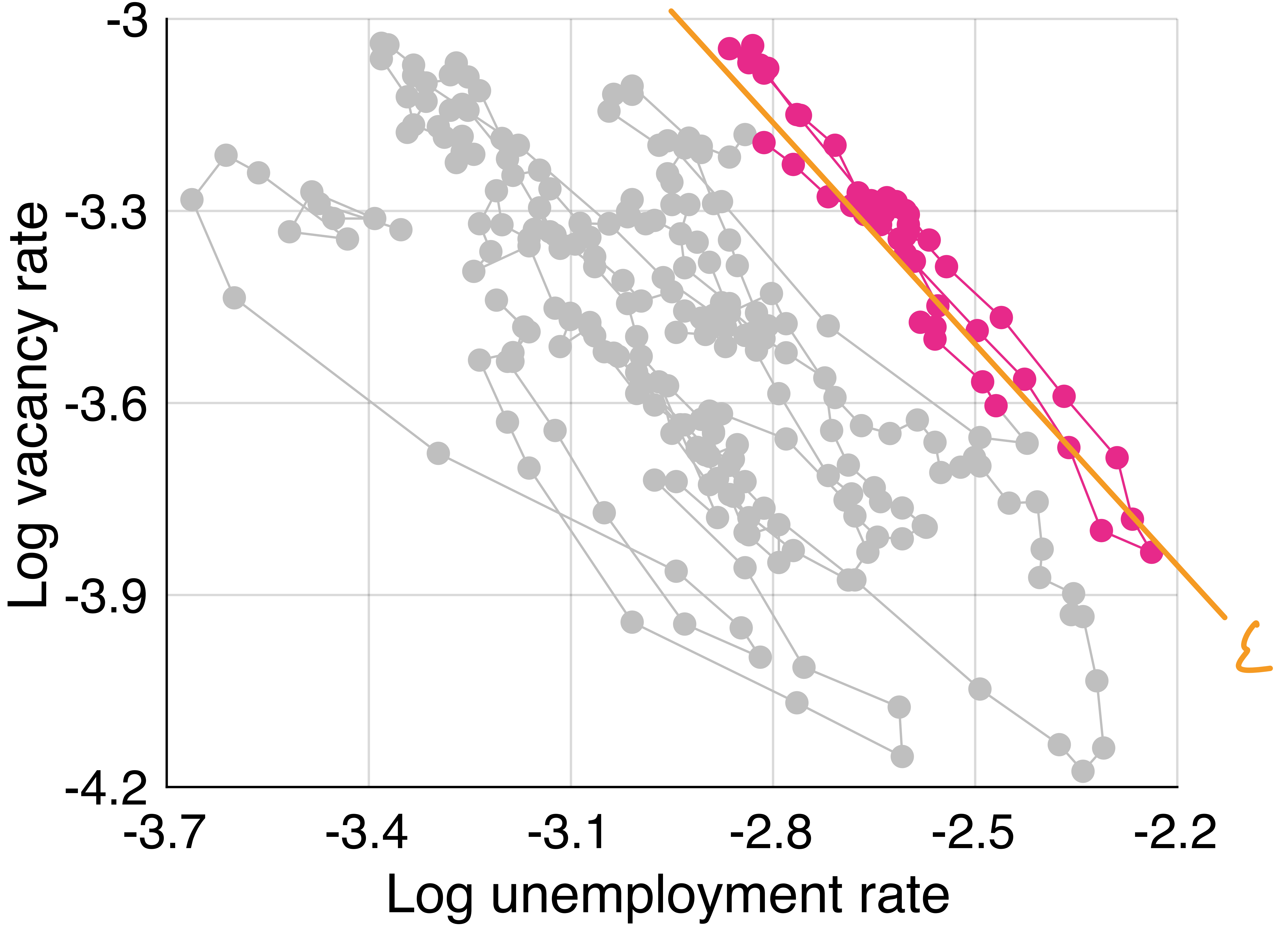

US since 1950s
ε
-3
-3.3
-3.6
-3.9
-4.2
Log vacancy rate
-3.7
-3.4
-3.1
-2.8
-2.5
-2.2
Log unemployment rate



-3
-3.3
-3.6
-3.9
-4.2
Log vacancy rate
-3.7
-3.4
-3.1
-2.8
-2.5
-2.2
Log unemployment rate
ε



-3
-3.3
-3.6
-3.9
-4.2
Log vacancy rate
-3.7
-3.4
-3.1
-2.8
-2.5
-2.2
Log unemployment rate
ε



-3
-3.3
-3.6
-3.9
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Log vacancy rate
-3.7
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-3.1
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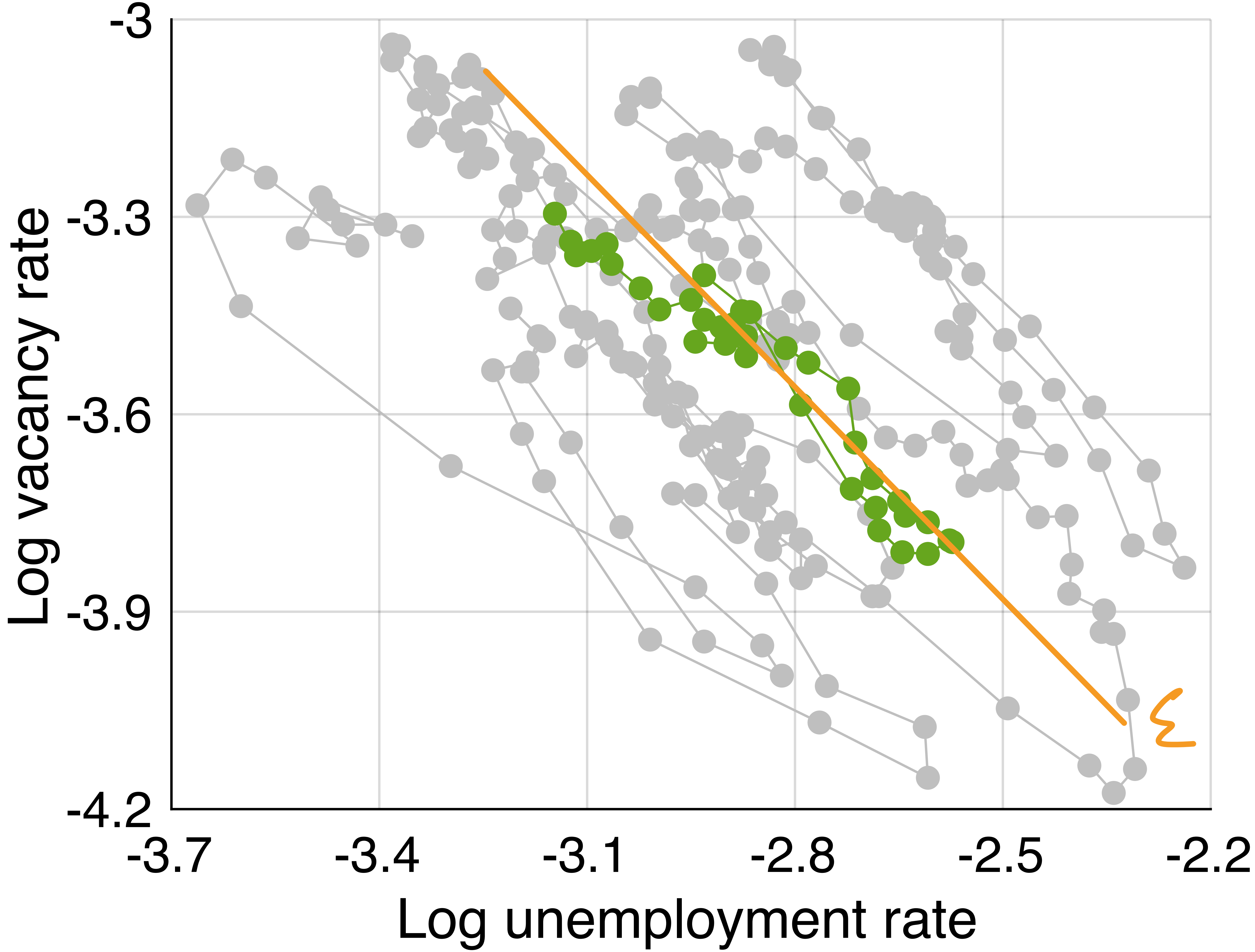
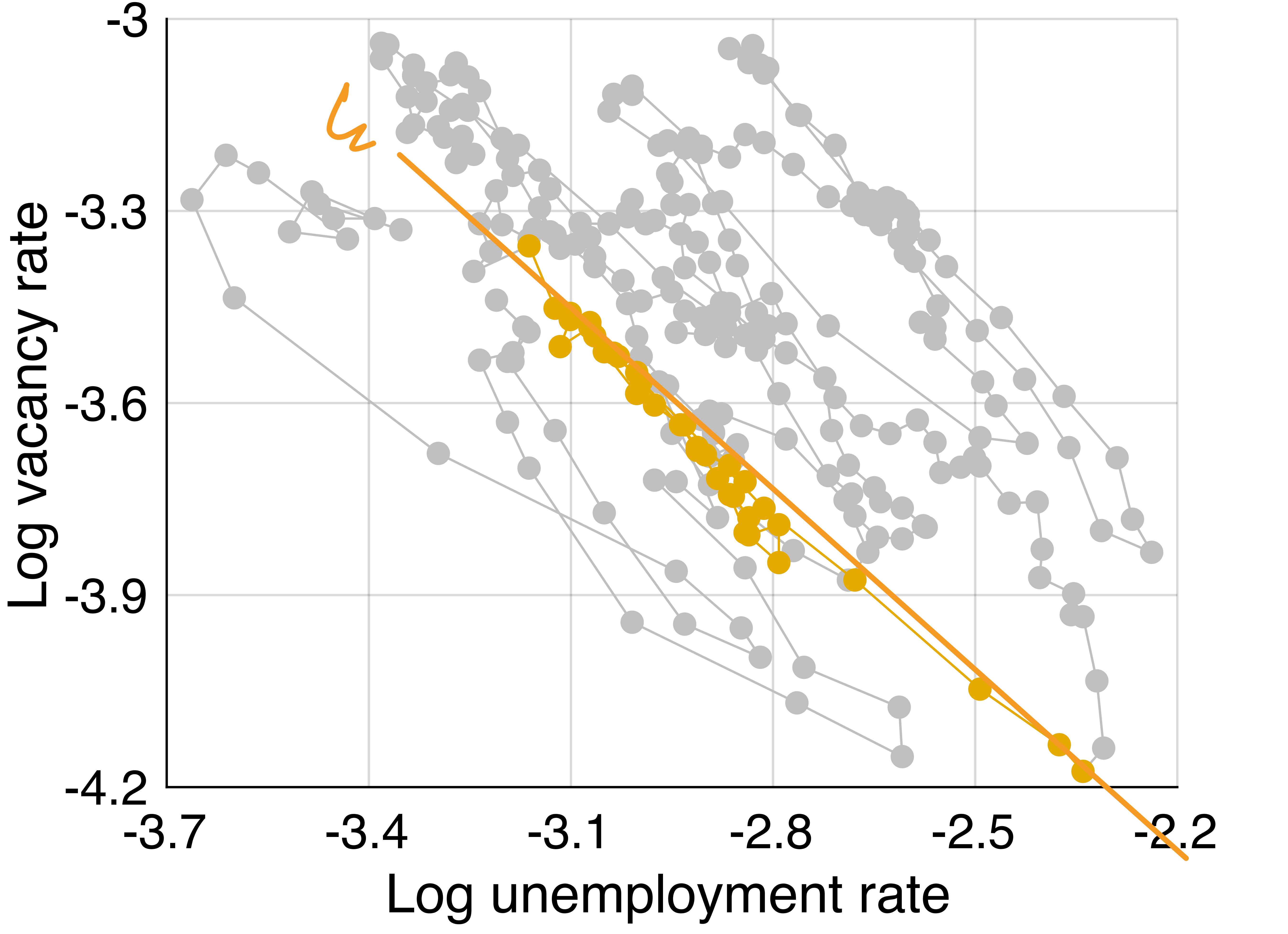
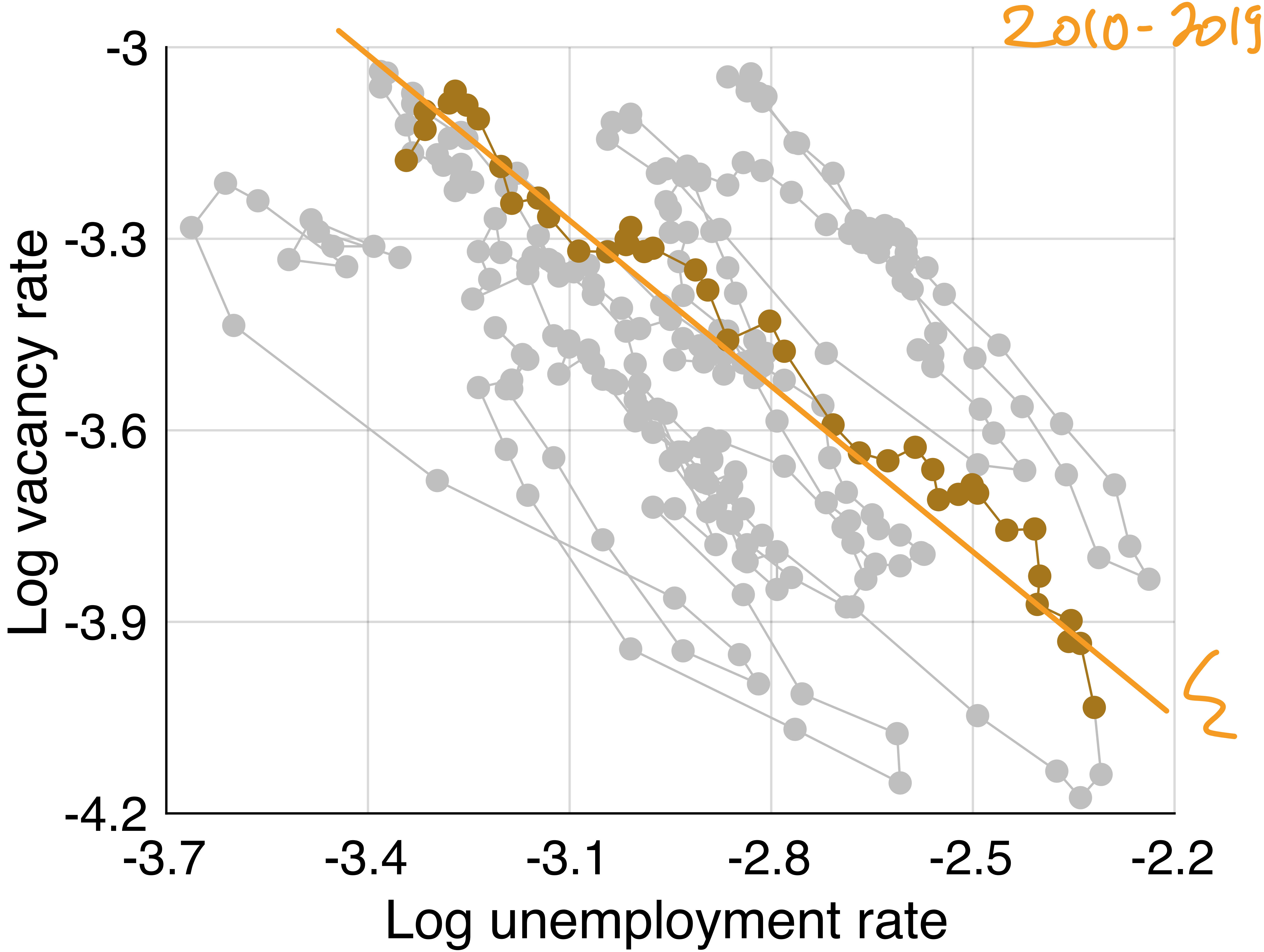
-2.2
Log unemployment rate



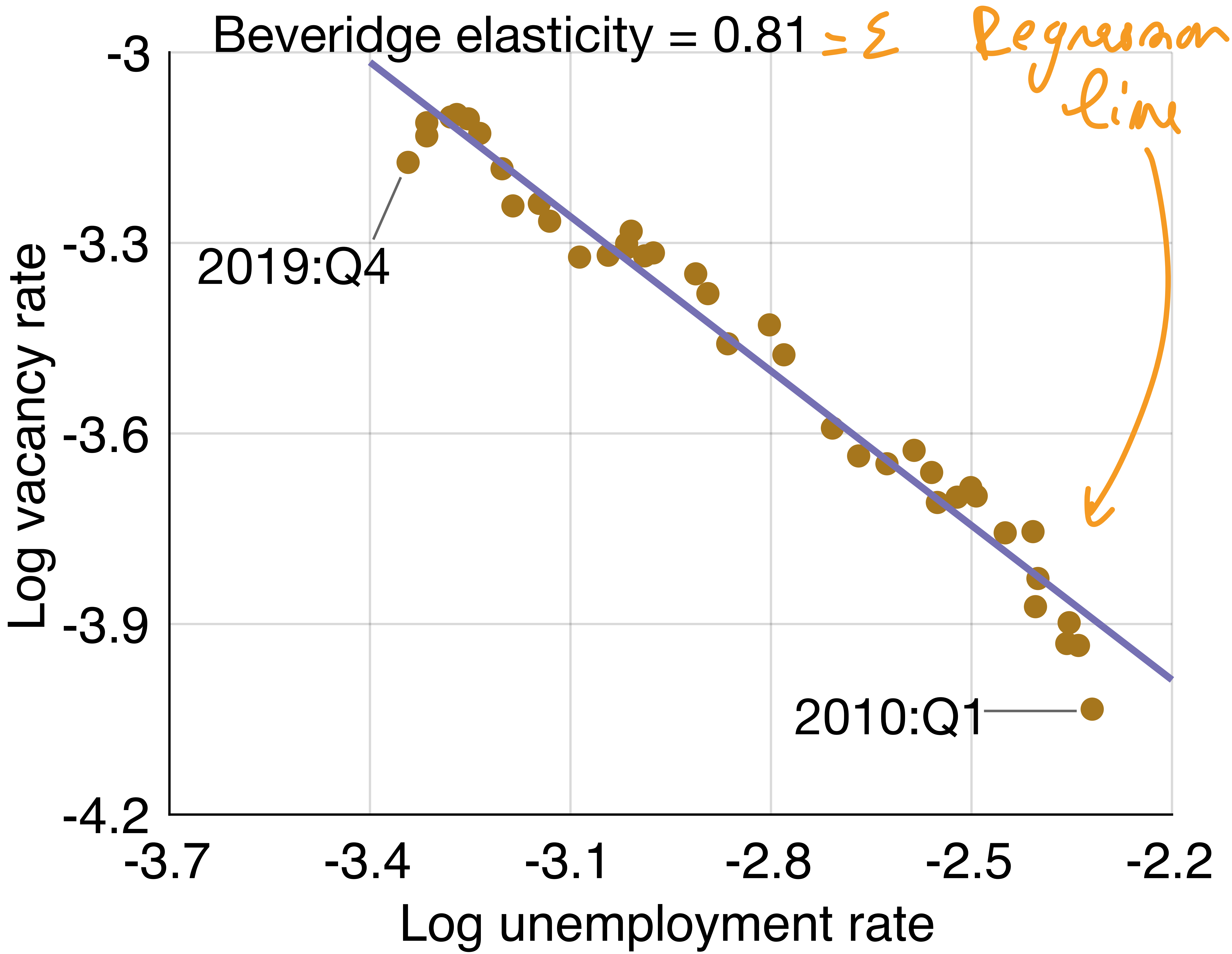
Log vacancy rate
-3
-3.3
-3.6
-3.9
-4.2
-3.7
-3.4
-3.1
-2.8
-2.5
-2.2
Log unemployment rate

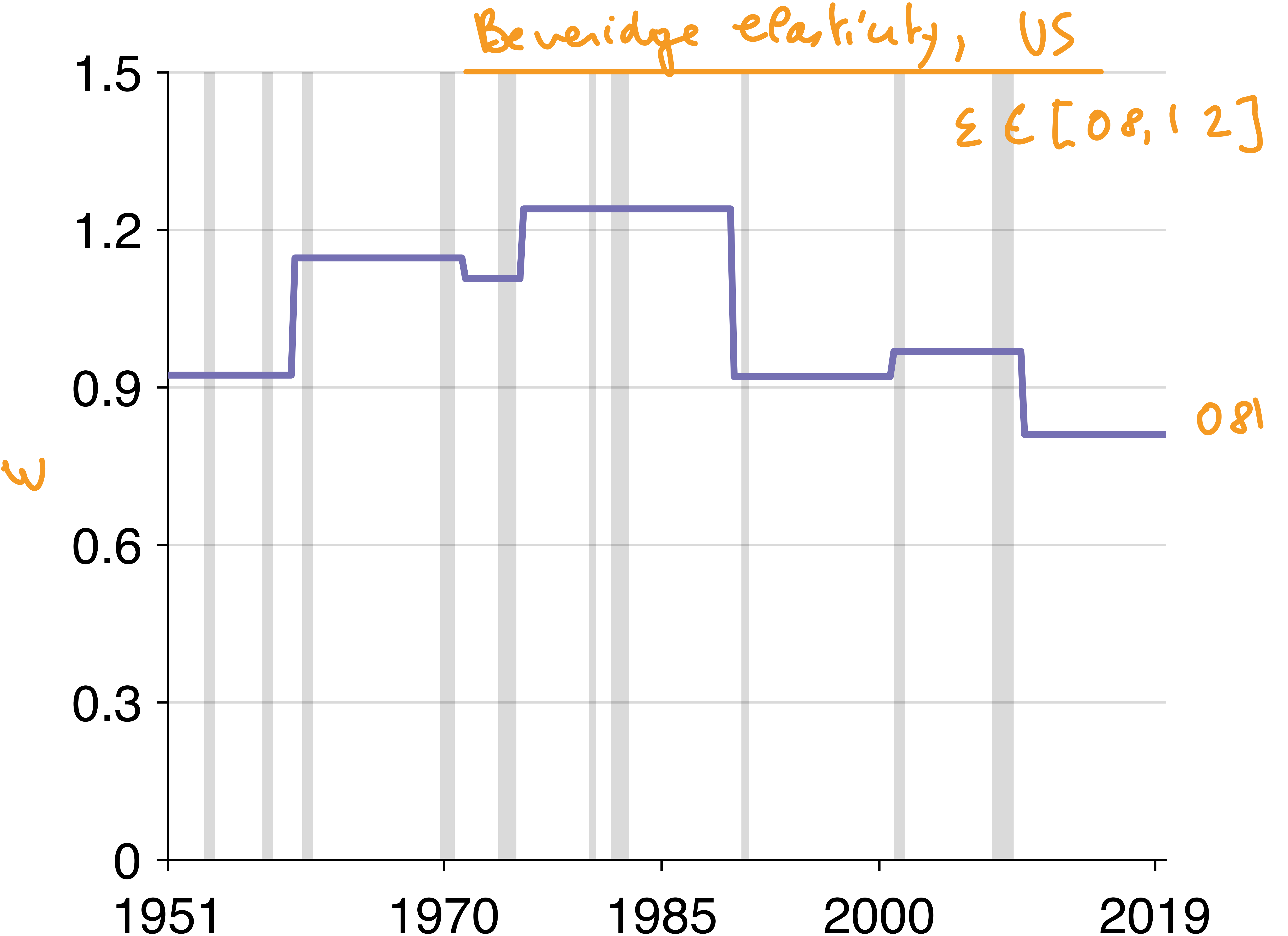


-3
-3.3
-3.6
-3.9
-4.2
-3.7
-3.4
-3.1
-2.8
-2.5
-2.2
Log vacancy rate
Log unemployment rate

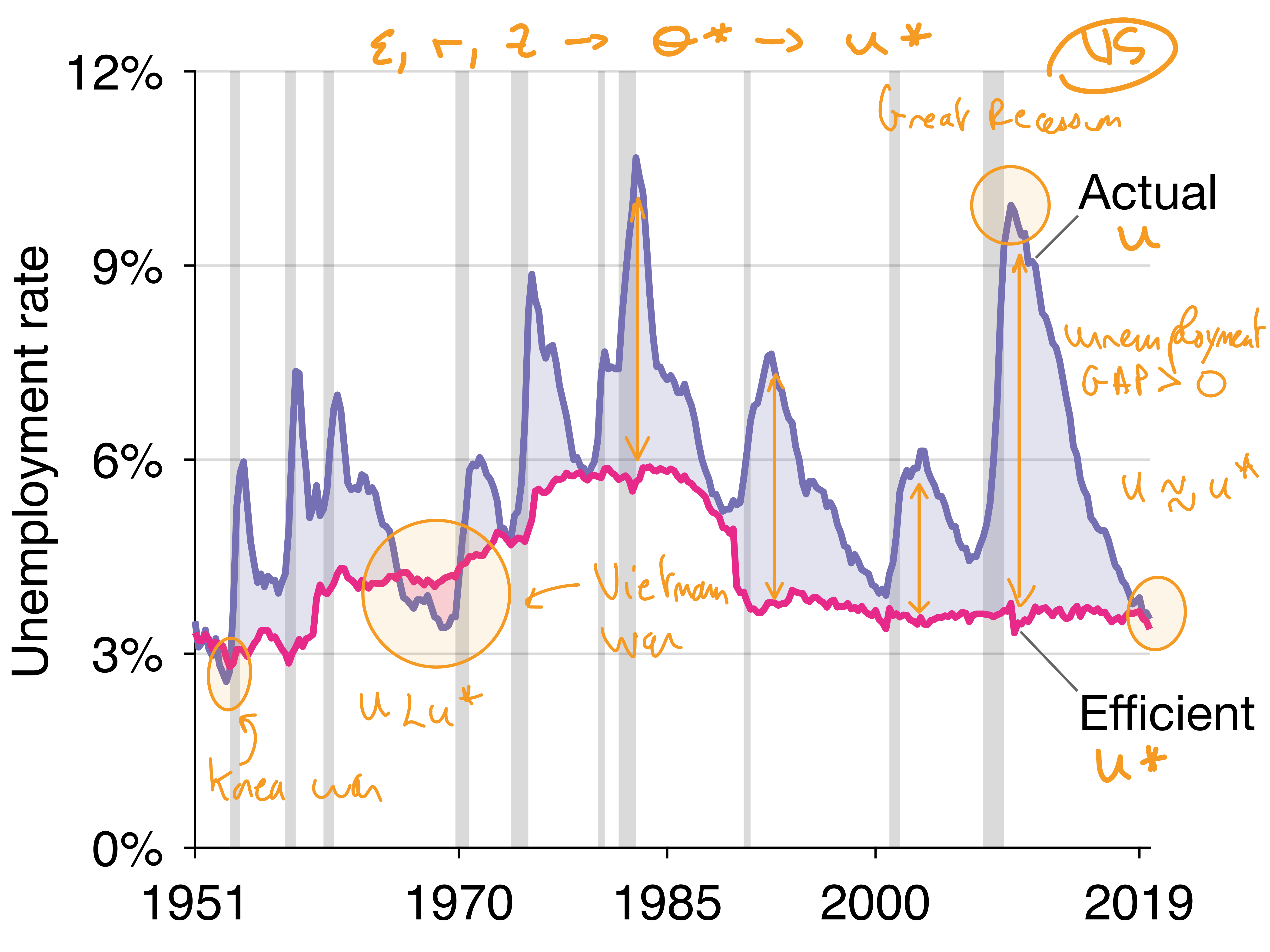


2010-2019
Log vacancy rate
-3
-3.3
-3.6
-3.9
-4.2
-3.7
-3.4
-3.1
-2.8
-2.5
-2.2
Log unemployment rate

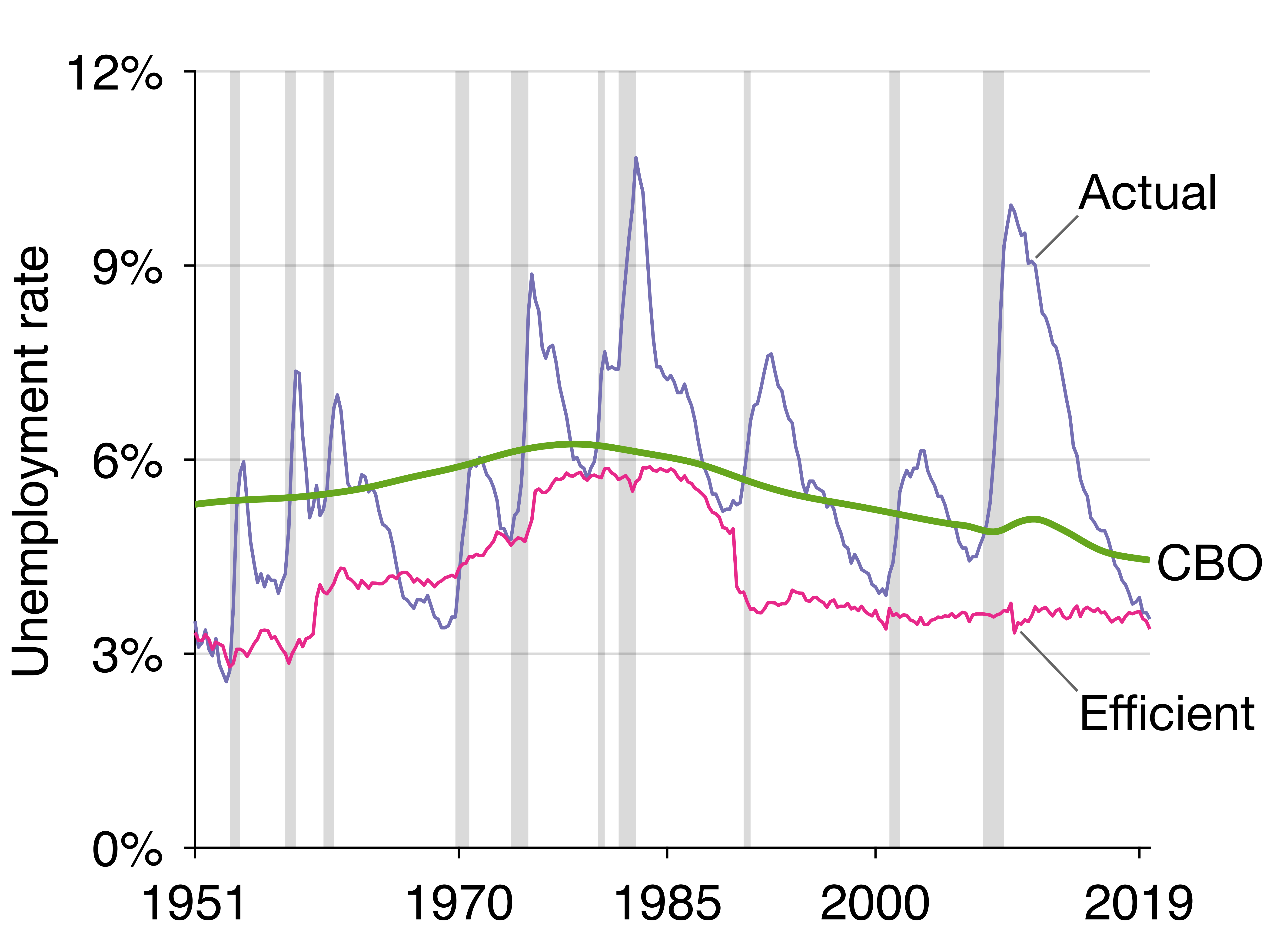

Beveridge elasticity = 0.81
= ε
Regression line
Log vacancy rate
Log unemployment rate
-3
-3.3
-3.6
-3.9
-4.2
-3.7
-3.4
-3.1
-2.8
-2.5
-2.2
2019:Q4
2010:Q1



Beveridge elasticity, US
ε ∈ [0.8, 1.2]
0.81
ε
1.5
1.2
0.9
0.6
0.3
0
1951
1970
1985
2000
2019



ε, r, z → θ* → u*
US
Great Recession
Actual
u
unemployment GAP > 0
u ≈ u*
Efficient
u*
Vietnam war
u < u*
Korea war
Unemployment rate
12%
9%
6%
3%
0%
1951
1970
1985
2000
2019



12%
9%
6%
3%
0%
Unemployment rate
1951
1970
1985
2000
2019
Actual
CBO
Efficient