Moen Meets Rotemberg: An Earthly Model of the Divine Coincidence

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This paper proposes a model of the divine coincidence, explaining its recent appearance in US data. The divine coincidence matters because it helps explain the behavior of inflation after the pandemic, and it guarantees that the full-employment and price-stability mandates of the Federal Reserve coincide. In the model, a Phillips curve relating unemployment to inflation arises from Moen (1997)'s directed search. The Phillips curve is nonvertical thanks to Rotemberg (1982)'s price-adjustment costs. The model's Phillips curve guarantees that the rate of inflation is on target whenever the rate of unemployment is efficient, generating the divine coincidence. If we assume that wage decreases—which reduce workers' morale—are more costly to producers than price increases—which upset customers—the Phillips curve also displays a kink at the point of divine coincidence.

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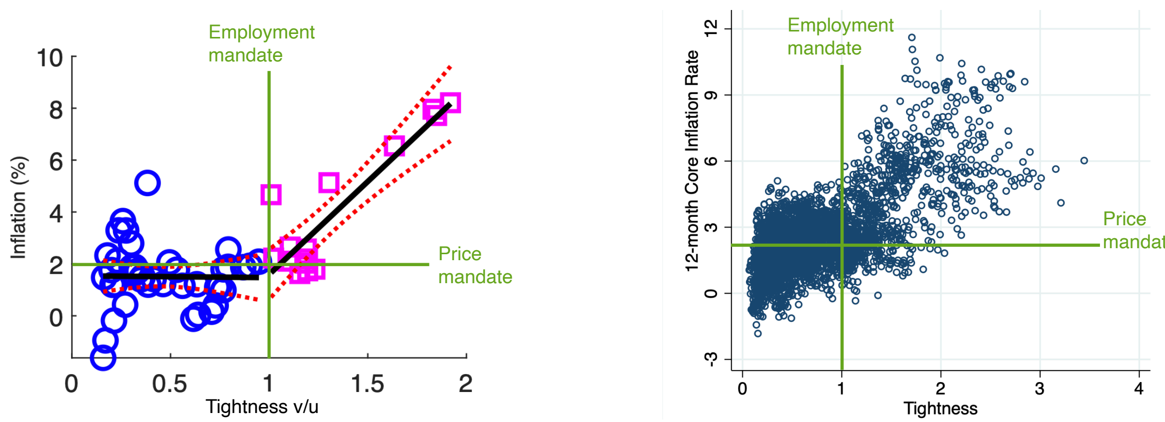
1. Introduction

Divine coincidence in the New Keynesian model. The textbook New Keynesian model features a divine coincidence: in the model, stabilizing inflation is equivalent to stabilizing the gap between the actual and efficient levels of output (Blanchard and Gali 2007). This property of the New Keynesian model is generally seen as unrealistic, so the model has been modified in several ways to remove it (Blanchard and Gali 2007, 2010).[1]

Divine coincidence in US data. A form of the divine coincidence seems to prevail in the United States, however. It appears that maintaining unemployment at its efficient level is equivalent to maintaining inflation on target (figure 1). That is, inflation is on target whenever the labor market is efficient; inflation rises above target whenever the labor market is inefficiently tight; and inflation falls below target whenever the labor market is inefficiently tight. The evidence of divine coincidence in the United States comes from the work of Benigno and Eggertsson (2023) and Gitti (2023). Benigno and Eggertsson use aggregate data for inflation and labor market tightness (the number of vacant jobs per unemployed workers). The divine coincidence appears clearly in the 2008–2022 period (figure 1A). When the labor market is efficient, which corresponds to a tightness of 1 (Michaillat and Saez 2023), inflation is on target at 2%. Gitti uses data at the metropolitan-area level for 2001–2022. The divine coincidence appears again at the metropolitan level, although in a more noisy fashion: when labor market tightness is 1, inflation is around 2% (figure 1B).

Implications of the divine coincidence. The divine coincidence has important implications. First, it helps explain the behavior of inflation. It says that inflation is on target whenever the labor market is efficient; rises above target whenever the labor market is inefficiently tight; and falls below target whenever the labor market is inefficiently slack. It can therefore explain part of the flare-up in inflation in 2021–2023, since the US labor market has been inefficiently tight in the aftermath of the coronavirus pandemic, from the middle of 2021 to today (Michaillat and Saez 2023). Second, it is important for policy. It implies that the full-employment and price-stability mandates of the Federal Reserve coincide. It means that by maintaining the economy at full employment, the Fed can be sure that inflation will also be on target.

[1] The divine coincidence also disappears from the New Keynesian model if trend inflation is nonzero (Alves 2014).



A. Aggregate data, 2008–2022

B. Metropolitan data, 2001–2022

FIGURE 1. Evidence of divine coincidence in the United States

Sources: Panel A was produced by Benigno and Eggertsson (2023, figure 4). Panel B was produced by Gitti (2023, figure 1). We added the lines marking the employment and price mandates to illustrate the divine coincidence.

Earthly model of the divine coincidence. This paper proposes a model of the divine coincidence. The model demonstrates that the divine coincidence arises in theory under fairly common assumptions. So it might not be as surprising as it first seemed that the divine coincidence appears in the data. More generally, the model developed here shows how the joint movements of inflation, unemployment, and tightness can be studied via the Euler, Phillips, and Beveridge curves.

Structure of the model. The model uses the structure of the economical business-cycle model developed by Michaillat and Saez (2022). In that model, however, inflation is constant. To generate price dynamics, we introduce price competition through directed search (Moen 1997). Furthermore, to ensure that unemployment fluctuates, we introduce price rigidity through quadratic price-adjustment costs (Rotemberg 1982). In the model, directed search produces a Phillips curve linking unemployment to inflation, and the price-adjustment costs ensure that the Phillips curve is nonvertical. The Phillips curve guarantees that inflation is on target whenever unemployment is efficient, generating the divine coincidence.

Kink in the Phillips curve. In the United States, it seems not only that the divine coincidence holds but also that the Phillips curve is kinked at the point of divine coincidence (Babb and Detmeister 2017; Smith, Timmermann, and Wright 2023; Benigno and Eggertsson 2023). The model is able to produce a kink in the Phillips curve at the point of divine coincidence by moving away from the assumption of symmetric price-adjustment costs. If we assume instead that wage decreases—which reduce workers' morale—are more costly to producers than price increases—which upset customers—the model generates a kinked Phillips curve. With this kink, the model predicts that macroeconomic shocks generate larger movements in inflation when the economy is inefficiently tight, and larger movements in unemployment when the economy is inefficiently slack.

Other applications of Moen-Rotemberg pricing. In this paper we combine Moen (1997)'s directed search with Rotemberg (1982)'s price-adjustment costs to generate a nonvertical Phillips curve. Such Moen-Rotemberg pricing could have applications in other models as well. Matching models—whether of the labor market, of the product market, or of the entire economy—require some price rigidity to generate realistic fluctuations in market tightness (Shimer 2005; Michaillat and Saez 2015). Various pricing mechanisms have been developed to generate such rigidity: fixed price norm (Hall 2005); credible bargaining (Hall and Milgrom 2008); staggered Nash bargaining (Gertler and Trigari 2009); and price norms that are rigid functions of the parameters (Blanchard and Gali 2010; Michaillat and Saez 2015). The Moen-Rotemberg pricing mechanism could also be used in any matching model that requires some price rigidity. A distinctive feature of Moen-Rotemberg pricing is that it relies on directed search, while the other rigid pricing mechanisms rely on random search.

2. Model

This section develops the model of the divine coincidence.

2.1. People

The size of the population is normalized to 1. People are organized in large households. The households are all initially identical and indexed by $j \in [0, 1]$. Household $j \in [0, 1]$ has l_j workers. The aggregate labor force is $l = \int_0^1 l_j(t)\, dj$.

2.2. Matching between workers and customers

Of the l_k workers of household k, y_{jk} work for household j, and a total $y_k = \int_0^1 y_{jk}(t)\, dk$ are employed across all households. Not all workers are employed, however. $U_k = l_k - y_k$

workers remain unemployed. The unemployment rate in household k is

$$u_k = \frac{U_k}{l_k}.$$

Services are sold through long-term worker-household relationships. Once a worker has matched with a household, she becomes a full-time employee of the household. She remains so until they separate, which occurs at rate $s > 0$.

To recruit workers from household k, household j sends V_{jk} of their own employees to visit household k's shop. These V_{jk} employees advertise the vacancies open by their employer, read applications from household k's workers, and interview and select suitable candidates. A total $V_k = \int_0^1 V_{jk}(t)\, dj$ employed workers are at shop k to recruit unemployed workers from household k. The recruiting rate at shop k is

$$v_k = \frac{V_k}{l_k}.$$

A matching function determines the flow of new matches at shop k based on the number of unemployed workers and recruiters: $h_k = h(U_k, V_k)$ where

$$h(U_k, V_k) = \omega \cdot \sqrt{U_k \cdot V_k} - s \cdot U_k. \tag{1}$$

The matching function h satisfies standard assumptions (Petrongolo and Pissarides 2001): it is 0 when $U = 0$ and $V = 0$, it has constant returns to scale, is increasing in V, and it is increasing in U as long as the market is not too slack—a condition that will be satisfied as long the unemployment rate is below 50%.

The partial derivative of the matching function with respect to U_k is

$$\frac{\partial h}{\partial U_k} = \frac{\omega}{2} \cdot \sqrt{\frac{V_k}{U_k}} - s = \frac{\omega}{2} \cdot \sqrt{\theta_k} - s,$$

where

$$\theta_k = \frac{V_k}{U_k} = \frac{v_k}{u_k}$$

is the tightness of market k. The partial derivative is positive for any $\theta_k \geq \underline{\theta}$ where the lower bound $\underline{\theta}$ is given by

$$\underline{\theta} = 4\left(\frac{s}{\omega}\right)^2. \tag{2}$$

We impose

$$\omega > 2s$$

so $\underline{\theta} < 1$. In the model the condition $\theta_k \geq \underline{\theta}$ is verified for any $u_k \in [0, 1/2]$. So as long the local unemployment rate is below 50%, the matching function will be increasing in the unemployment rate. The matching function is also concave in U and concave in V.

Although it satisfies all standard properties, the matching function h takes an unusual form. We specify the function as such to obtain an hyperbolic Beveridge curve in the model, and therefore be consistent with the empirical Beveridge curve observed in US data (Michaillat and Saez 2023).

The tightness on market k is the ratio of the number of recruiters (buyers) and unemployed workers (sellers):

$$\theta_k = \frac{V_k}{U_k}.$$

Equivalently, tightness on market k is the ratio of the recruiting and unemployment rates: $\theta_k = v_k/u_k$. Tightness on market k is also the number of recruiters per unemployed workers. We impose $\theta_k \geq \underline{\theta}$ to ensure that the matching function is increasing in U, which also ensures that the number of matches is positive.

Tightness determines all trading rates. The customer-finding rate is

$$f(\theta_k) = \frac{h_k}{U_k} = \omega \cdot \sqrt{\theta_k} - s. \tag{3}$$

For $\theta_k \in [\underline{\theta}, \infty)$, the customer-finding rate f is positive since $\sqrt{\theta_k} \geq 2s/\omega$ so $\omega\sqrt{\theta_k} - s \geq s > 0$. The customer-finding rate is also increasing in θ_k. In fact, $f(\underline{\theta}) = s$ and $f(\infty) = \infty$. Hence, when tightness is higher, it is easier to find a job and sell services.

The worker-finding rate is

$$q(\theta_k) = \frac{h_k}{V_k} = \frac{\omega}{\sqrt{\theta_k}} - \frac{s}{\theta_k}. \tag{4}$$

For $\theta_k \in [\underline{\theta}, \infty)$, the worker-finding rate is positive since $\sqrt{\theta_k} \geq 2s/\omega$ so $s/\sqrt{\theta_k} \leq \omega/2$ and $\omega - s/\sqrt{\theta_k} \geq \omega/2 > 0$. The worker-finding rate is also decreasing in θ_k.

The derivative of the worker-finding rate with respect to θ_k is

$$\frac{dq}{d\theta_k} = -\frac{\omega}{2} \cdot \theta_k^{-3/2} + s\theta_k^{-2} = -\theta_k^{-3/2} \cdot \left[\frac{\omega}{2} - \frac{s}{\sqrt{\theta_k}}\right] < 0.$$

Indeed, we showed that for $\theta_k \in [\underline{\theta}, \infty)$,

$$\omega - \frac{s}{\sqrt{\theta_k}} \geq \frac{\omega}{2},$$

so that

$$\frac{\omega}{2} - \frac{s}{\sqrt{\theta_k}} \geq 0,$$

which implies that $dq/d\theta_k < 0$. In fact, $q(\underline{\theta}) = \omega^2/4s > s$ and $q(\infty) = 0$. This means that when tightness is higher, it is harder to find a worker and buy services.

Note also that as usual, $f(\theta_k) = \theta_k \cdot q(\theta_k)$.

2.3. Cost of unemployment and hiring

Unemployed workers wait in their shop to be hired. During that time, they do not receive any income and cannot engage in home production, which explains why unemployment is individually and socially costly.

There are costs not only on the selling side of the market, but also on the buying side. Hiring workers is indeed costly. Each recruiter looking to hire a worker on behalf of their employer cannot produce utility-providing services. Because the recruiters employed by household j to hire workers from household k do not provide direct utility but are used in the process of hiring other workers, consumption c_{jk} is less than output y_{jk}. Consumption c_{jk} is the number of workers from household k working for household j, minus the number of workers employed by household j to recruit workers from household k:

$$c_{jk} = y_{jk} - V_{jk}.$$

2.4. Balanced flows and unemployment

We assume that flows on each individual market are balanced. This assumption is motivated by the fact that on the US labor market, flows are always approximately balanced (Michaillat and Saez 2021a, p. 7).

The number of workers employed in household k is given by a differential equation:

$$\dot{y}_k = f(\theta_k) \cdot U_k - s \cdot y_k = f(\theta_k) \cdot U_k - s \cdot [l_k - U_k] = l_k \cdot \left[f(\theta_k) \cdot u_k - s \cdot [1 - u_k]\right].$$

We assume that flows are balanced in all (j, k) cells. In particular flows are balanced in household k: $\dot{y}_k = 0$. This assumption implies that the local unemployment rate is a function of local tightness: $u_k = u(\theta_k)$, where

$$u(\theta_k) = \frac{s}{s + f(\theta_k)} = \frac{s/\omega}{\sqrt{\theta_k}}. \tag{5}$$

From this expression we see that for any $\theta_k \geq \underline{\theta} = 4(s/\omega)^2$, $u(\theta_k) \leq 1/2$. The analysis focuses on this range of tightness—which is without real loss of generality since in practice unemployment rates are not above 50%.

The unemployment function (5) has the following properties when $\theta_k \in [\underline{\theta}, \infty)$: $u(\underline{\theta}) = 1/2$, $u(\infty) = 0$, and u is decreasing in θ_k. When the market is tighter, workers find jobs more rapidly, so the unemployment rate is lower.

Thanks to the shape of the matching function (1), the unemployment rate is an isoelastic function of tightness, and the Beveridge curve is isoelastic as well. In fact the Beveridge curve is a rectangular hyperbola, just like in US data (Michaillat and Saez 2023):

$$v(u_k) = \frac{(s/\omega)^2}{u_k}. \tag{6}$$

From the Beveridge curve we can also express the recruiting rate as a function of local tightness:

$$v(\theta_k) = \frac{s}{\omega} \cdot \sqrt{\theta_k}. \tag{7}$$

The recruiting function (7) has the following properties when $\theta_k \in [\underline{\theta}, \infty)$: $v(\underline{\theta}) = 2(s/\omega)^2$, $v(\infty) = \infty$, and v is increasing in θ_k. When the market is tighter, it takes longer to hire workers, so the recruiting rate is higher.

2.5. Balanced flows and recruiter-producer ratio

Next we compute the recruiter-producer ratio for household k. The number of employed workers from household j in household k follows a differential equation:

$$\dot{y}_{jk} = q(\theta_k) \cdot V_{jk} - s \cdot y_{jk} = q(\theta_k) \cdot [y_{jk} - c_{jk}] - s \cdot y_{jk}.$$

We assume that flows are balanced in all (j, k) cells, so $\dot{y}_{jk} = 0$. This means that tightness determines the gap between consumption and output:

$$y_{jk} = [1 + \tau(\theta_k)] \cdot c_{jk},$$

where the recruiter-producer ratio is a function of tightness:

$$\tau(\theta_k) = \frac{s}{q(\theta_k) - s}. \tag{8}$$

By definition, $\tau(\theta_k) = (y_{jk} - c_{jk})/c_{jk}$. The quantity $y_{jk} - c_{jk}$ is the number of workers hired by household j to recruit new workers from household k, while c_{jk} is the number of workers from household k hired by household j for producing services. Hence the function τ gives the recruiter-producer ratio required for any local tightness θ_k.

Given the properties of the worker-finding rate q, we infer the properties the properties of the recruiter-producer ratio τ. Since $q(\underline{\theta}) = \omega^2/4s$, $\tau(\underline{\theta}) = 1/[(\omega/2s)^2 - 1] = 1/(1/\underline{\theta} - 1) > 0$. Since q is decreasing in θ for $\theta \geq \underline{\theta}$, τ is increasing in θ for $\theta \geq \underline{\theta}$. Furthermore, $\tau \to \infty$ when $\theta \to \bar{\theta}$ when $\bar{\theta} > \underline{\theta}$ is defined by $q(\bar{\theta}) = s$. The upper tightness bound is well defined because $q(\underline{\theta}) > s$, q is decreasing in θ, and $q(\infty) \to 0$. In fact it is possible to express $\bar{\theta}$ as a function of $\underline{\theta}$ alone (appendix A):

$$\bar{\theta} = \frac{\underline{\theta}}{\left[1 - \sqrt{1 - \underline{\theta}}\right]^2} > \underline{\theta}.$$

It is also helpful to write the recruiter-producer ratio as a function of the unemployment rate. In a local market k, the recruiter-producer $\tau(\theta_k)$ ratio is the same in all households that hire workers there. So the recruiter-producer ratio is all the ratio between all the recruiters hired from household k and all the producers from household k:

$$\tau_k = \frac{y_k - c_k}{c_k}, \qquad y_k - c_k = v_k, \qquad c_k = l_k - u_k - v_k.$$

Therefore we can write the recruiter-producer as a function of the unemployment rate:

$$\tau(u_k) = \frac{v(u_k)}{1 - [u_k + v(u_k)]}. \tag{9}$$

2.6. Some elasticities

We now compute a few elasticities that will be important when we solve the model.

The elasticity of the customer-finding rate f given by (3) is

$$\frac{d\ln(f)}{d\ln(\theta)} = \frac{\omega\sqrt{\theta}}{\omega\sqrt{\theta}-s}\cdot\frac{1}{2} = \frac{1}{1-(s/\omega)/\sqrt{\theta}}\cdot\frac{1}{2} = \frac{1/2}{1-u},$$

where the unemployment rate is a function of tightness given by (5). Since $1-u\approx 1$, the elasticity is never far from 1/2, as it would be with a more common Cobb-Douglas matching function.

Since $q(\theta) = f(\theta)/\theta$, we infer the elasticity of the worker-finding rate q:

$$\frac{d\ln(q)}{d\ln(\theta)} = \frac{d\ln(f)}{d\ln(\theta)} - 1 = -\frac{1/2-u}{1-u},$$

where the unemployment rate is a function of tightness given by (5). Since $1-u\approx 1$ and $1/2-u\approx 1/2$, the elasticity is never far from $-1/2$, as it would be with a more common Cobb-Douglas matching function.

The elasticity of the unemployment rate (5) simply is

$$\frac{d\ln(u)}{d\ln\left(\theta_k\right)} = -(1-u)\cdot\frac{d\ln(f)}{d\ln(\theta)} = -\frac{1}{2}. \tag{10}$$

The elasticity of the recruiter-producer ratio (8) is given by

$$\frac{d\ln(\tau)}{d\ln\left(\theta_k\right)} = -(1+\tau)\cdot\frac{d\ln(q)}{d\ln(\theta)} = \frac{(1+\tau)\cdot(1/2-u)}{1-u}. \tag{11}$$

From (9), we have

$$1+\tau(\theta_k) = \frac{1-u(\theta_k)}{1-u(\theta_k)-v(\theta_k)},$$

so we can simplify the elasticity of the recruiter-producer ratio:

$$\frac{d\ln(\tau)}{d\ln\left(\theta_k\right)} = \frac{1}{2}\cdot\frac{1-2u}{1-u-v}. \tag{12}$$

2.7. Productive efficiency at shop k

What is the efficient allocation of labor at shop k? We are interested in productive efficiency, that is the allocation of labor that maximizes the amount of services from household k that are consumed. The amount of services consumed is

$$c_k = y_k - V_k = l_k - U_k - V_k = l_k \cdot \left[1 - u_k - v_k\right].$$

Maximizing that amount is equivalent to minimizing the sum of the unemployment and recruiting rates, $u_k + v_k$, subject to the Beveridge curve (6). This is exactly the problem studied by Michaillat and Saez (2023). The solution is

$$u_k^* = \sqrt{u_k v_k} = s/\omega, \qquad v_k^* = u_k^*, \qquad \theta_k^* = 1. \tag{13}$$

Furthermore, the economy is inefficiently tight whenever there are more recruiters than jobseekers, $u_k < v_k$, inefficiently slack whenever there are more jobseekers than recruiters, $u_k > v_k$, and of course efficient whenever there are as many jobseekers as recruiters, $u_k = v_k$.

2.8. Directed search and price-tightness competition

All workers from household k charge a price p_k per unit time. The expenditure by household j on workers k therefore is

$$p_k \cdot y_{jk} = p_k \cdot [1 + \tau(\theta_k)] \cdot c_{jk}.$$

The relevant price of services is not just p_k but $p_k \cdot [1 + \tau(\theta_k)]$. The price involves the price per unit time as well as the time it takes to replace a worker.

All workers are perfectly substitutable, so households hires workers from the household that offers the cheapest consumption. All households are aware of this fact, so all households price their services to compete with other households:

$$p_k \cdot [1 + \tau(\theta_k)]$$

must be the same across all households k. Just as in Moen (1997), buyers direct their search toward the most attractive sellers, which induces competition across all sellers. Through competition, sellers set prices so buyers are indifferent across all sellers. If sellers set a higher price, then cheaper workers would be available, or workers could be hired with less wait, so they would not get any customers.

Accordingly, there is a price level p such that for all k,

$$p_k \cdot [1 + \tau(\theta_k)] = p \cdot [1 + \tau(\theta)], \tag{14}$$

where the aggregate market tightness is the ratio of the aggregate number of recruiters to the aggregate number of unemployed workers, given by

$$\theta = \frac{\sum_k V_k}{\sum_k U_k}.$$

2.9. Effect of local price on local tightness

The price chosen by household j determines the tightness θ_j it faces, and therefore the pace at which workers from the household find employment. From (14), we see that the local tightness is given by

$$\theta_j(p_j) = \tau^{-1}\left(\frac{p}{p_j}[1 + \tau(\theta)] - 1\right).$$

The function τ^{-1} is increasing, so the local tightness $\theta_j(p_j)$ is decreasing in the local price p_j. A high price leads to low tightness and high unemployment. A low price leads to high tightness and low unemployment. In that way, households face downward-sloping demand curves in a price-tightness plane.

In fact, the demand curve $\theta_j(p_j)$ has the following properties for $p_j \in (0, p[1 + \tau(\theta)])$: $\theta_j(0) = \bar{\theta}$, $\theta_j(p) = \theta$, $\theta_j(p[1 + \tau(\theta)]) = 0$. The derivative and elasticity of the demand curve are:

$$\frac{d\theta_j}{dp_j} = -\frac{p}{p_j^2} \cdot [1 + \tau(\theta)] \cdot \frac{1}{\tau'(\theta_j)} = -\frac{1 + \tau(\theta_j)}{p_j \cdot \tau'(\theta_j)}$$

$$\frac{d\ln\left(\theta_j\right)}{d\ln\left(p_j\right)} = -\frac{1 + \tau(\theta_j)}{\theta_j \cdot \tau'(\theta_j)} = \frac{-1}{d\ln(1 + \tau(\theta_j))/d\ln(\theta_j)}.$$

From (11), we infer that

$$\frac{d\ln(1 + \tau)}{d\ln(\theta)} = \frac{\tau}{1 + \tau}\frac{d\ln(\tau)}{d\ln(\theta)} = \frac{\tau \cdot (1/2 - u)}{1 - u}.$$

Hence the elasticity of the demand curve is

(15)
$$\frac{d\ln\left(\theta_j\right)}{d\ln\left(p_j\right)} = -\frac{1-u(\theta_j)}{\tau(\theta_j)\cdot[1/2-u(\theta_j)]}.$$

2.10. Efficiency without price-adjustment costs

As a benchmark, we consider the case without any price-adjustment cost. In that case, seller k is free to set any price she wants to maximize labor income. That is, she choose p_k to maximize $p_k \cdot y_k$ subject to the demand constraint (14). Because of the demand constraint, labor income can be written

$$p_k \cdot y_k = p\cdot[1+\tau(\theta)]\cdot\frac{y_k}{1+\tau(\theta_k)} = p\cdot[1+\tau(\theta)]\cdot\frac{1-u(\theta_k)}{1+\tau(\theta_k)}\cdot l_k.$$

The variables τ, u, v are linked by (9), so

$$\frac{1-u(\theta_k)}{1+\tau(\theta_k)} = 1-u(\theta_k)-v(\theta_k).$$

Accordingly, seller k sets local tightness θ_k so as to minimize $u(\theta_k)+v(\theta_k)$. This is equivalent to choosing the unemployment rate u_k so as to minimize $u_k+v(u_k)$, where the unemployment and recruiting rates are related by the Beveridge curve (6). The local tightness θ_k and unemployment rate u_k are therefore chosen efficiently: (13) holds, so that in particular $\theta_k = 1$. Here we have just recovered the central efficiency result of Moen (1997).

2.11. Price rigidity

Generally, tightness and unemployment rate are not efficient because prices are somewhat rigid. The local inflation for household k is

(16)
$$\pi_k(t) = \frac{\dot{p}_k(t)}{p_k(t)}.$$

Changing prices is costly. As in Rotemberg (1982), households incur a quadratic price-adjustment cost when local inflation departs from normal inflation $\bar{\pi}$. The flow disutility caused by prices deviating from the norm is

$$\rho(\pi_k) = \frac{\kappa}{2} \cdot [\pi_k - \bar{\pi}]^2.$$

This quadratic cost appears in the household's utility function.

2.12. People's preferences

People care about two things: their consumption of services and their social status, measured by their relative wealth. In addition people incur a cost from price changes. Each household maximizes the discounted sum of flow utilities,

$$\int_0^{\infty} e^{-\delta t} \left\{ \ln\left(c_j(t)\right) + \sigma \cdot \left[\frac{b_j(t)}{p(t)} - \frac{b(t)}{p(t)}\right] - \frac{\kappa}{2} \cdot [\pi_j - \bar{\pi}]^2 \right\} dt,$$

where $\delta > 0$ is the time discount rate, $\sigma > 0$ indicates concerns for social status, $c_j(t) = \int_0^1 c_{jk}(t)\, dk$ is total consumption of services, $b_j(t)$ is saving in government bonds, and $b(t) = \int_0^1 b_j(t)\, dj$ is aggregate wealth in the economy.

2.13. People's budget constraint

People are subject to a budget constraint. This constraint takes the form of a law of motion of government bond holdings. For household j, the law of motion is

$$\dot{b}_j = i \cdot b_j - \int_0^1 p_k y_{jk}\, dk + p_j y_j.$$

Because of the matching process and the equalization of prices achieved through directed search, the household's expenditure on services can be rewritten as follows:

$$\int_0^1 p_k y_{jk}\, dk = \int_0^1 p_k [1 + \tau(\theta_k)] c_{jk}\, dk = p \cdot [1 + \tau(\theta)] \cdot \int_0^1 c_{jk}\, dk = p \cdot [1 + \tau(\theta)] \cdot c_j.$$

Then, because of the matching process, the household's income becomes

$$p_j \cdot y_j = p_j \cdot [1 - u(\theta_j(p_j))] \cdot l_j.$$

Accordingly, the law of motion can be written as

$$\dot{b}_j = i \cdot b_j - p \cdot [1 + \tau(\theta)] \cdot \int_0^1 c_{jk}\, dk + p_j \cdot [1 - u(\theta_j(p_j))] \cdot l_j. \tag{17}$$

3. Model solution

We now solve the model. The main step is to solve household j's maximization problem, which we do by Hamiltonian.

3.1. Construction of the Hamiltonian

The Hamiltonian of household j's problem is

$$\begin{aligned}\mathcal{H}_j = \ln\left(c_j\right) + \sigma \cdot \left[\frac{b_j}{p} - \frac{b}{p}\right] - \frac{\kappa}{2} \cdot [\pi_j - \bar{\pi}]^2 \\ + \mathcal{A}_j \cdot \left[i \cdot b_j - p \cdot [1 + \tau] \cdot c_j + p_j \cdot [1 - u(\theta_j(p_j))] \cdot l_j\right] \\ + \mathcal{B}_j \cdot \pi_j \cdot p_j.\end{aligned}$$

The control variables are consumption c_j and inflation π_j. The state variables are bond holdings b_j and price level p_j. The costate variables are $\mathcal{A}_j$, which applies to the law of motion of bond holdings (17), and $\mathcal{B}_j$, which applies to the law of motion of the price level (16).

We focus on a symmetric solution of model, in which all households behave the same. In this symmetric situation, we can drop the index j.w

3.2. First-order condition with respect to consumption

We begin with the first-order condition $d\mathcal{H}_j/dc_j = 0$. It gives

$$1/c_j = \mathcal{A}_j \cdot p \cdot [1 + \tau]$$

$$1/\mathcal{A} = p \cdot [1 + \tau] \cdot c$$

$$1/\mathcal{A} = p \cdot y. \tag{18}$$

Taking the log and then time derivative of this last equation yields

$$-\ln(\mathcal{A}) = \ln(p) + \ln(y)$$

$$-\frac{\dot{\mathcal{A}}}{\mathcal{A}} = \pi + \frac{\dot{y}}{y}. \tag{19}$$

3.3. First-order condition with respect to inflation

Next we turn to the first-order condition $d\mathcal{H}_j/d\pi_j = 0$. It yields

$$\mathcal{B}_j \cdot p_j = \kappa \cdot (\pi_j - \bar{\pi})$$

$$\mathcal{B} = \frac{\kappa}{p} \cdot (\pi - \bar{\pi}). \tag{20}$$

Taking the log and then time derivative of the last equation yields

$$\ln(\mathcal{B}) = \ln(\kappa) - \ln(p) + \ln(\pi - \bar{\pi}).$$

$$\frac{\dot{\mathcal{B}}}{\mathcal{B}} = -\pi + \frac{\dot{\pi}}{\pi - \bar{\pi}}. \tag{21}$$

3.4. First-order condition with respect to saving

The next first-order condition is $d\mathcal{H}_j/db_j = \delta \cdot \mathcal{A}_j - \dot{\mathcal{A}}_j$. It gives

$$\frac{\sigma}{p} + \mathcal{A}_j \cdot i = \delta \cdot \mathcal{A}_j - \dot{\mathcal{A}}_j.$$

Reshuffling the terms yields

$$\frac{\dot{\mathcal{A}}}{\mathcal{A}} = \delta - i - \frac{\sigma}{p \cdot \mathcal{A}}$$

Using $1/(p \cdot \mathcal{A}) = y$, which comes from (18), we obtain

$$\frac{\dot{\mathcal{A}}}{\mathcal{A}} = \delta - (i + \sigma \cdot y) \tag{22}$$

3.5. First-order condition with respect to price

The final first-order condition is $d\mathcal{H}_j/dp_j = \delta \cdot \mathcal{B}_j - \dot{\mathcal{B}}_j$. This condition becomes

$$\mathcal{A}_j \cdot (1 - u_j) \cdot l_j - \mathcal{A}_j \cdot p_j \cdot l_j \cdot u'(\theta_j) \cdot \theta'(p_j) + \mathcal{B}_j \cdot \pi_j = \delta \cdot \mathcal{B}_j - \dot{\mathcal{B}}_j.$$

From the elasticity (10), we have the following derivative:

$$u'(\theta_j) = -\frac{u(\theta_j)}{2 \cdot \theta_j}.$$

And from the elasticity (15), we have

$$\theta'(p_j) = -\frac{\theta_j[1 - u(\theta_j)]}{\tau(\theta_j) \cdot p_j \cdot [1/2 - u(\theta_j)]}.$$

Hence,

$$p_j \cdot u'(\theta_j) \cdot \theta'(p_j) = \frac{1 - u(\theta_j)}{1 - 2u(\theta_j)} \cdot \frac{u(\theta_j)}{\tau(\theta_j)}.$$

Reshuffling terms gives:

$$(\delta - \pi_j) \cdot \mathcal{B}_j - \dot{\mathcal{B}}_j = \mathcal{A}_j \cdot y_j \cdot \left[1 - \frac{u(\theta_j)}{\tau(\theta_j)[1 - 2u(\theta_j)]}\right]$$

$$-\frac{\dot{\mathcal{B}}}{\mathcal{B}} = \pi - \delta + \frac{\mathcal{A} \cdot y}{\mathcal{B}} \cdot \left[1 - \frac{u(\theta)}{\tau[1 - 2u(\theta)]}\right].$$

Using $y \cdot \mathcal{A} = 1/p$, which comes from (18), and $\mathcal{B} = \kappa(\pi_j - \bar{\pi})/p$, which comes from (20), we link inflation to the unemployment rate:

$$-\frac{\dot{\mathcal{B}}}{\mathcal{B}} = \pi - \delta + \frac{1}{\kappa} \cdot \frac{1}{\pi - \bar{\pi}} \cdot \left[1 - \frac{u}{\tau[1 - 2u]}\right].$$

Then, using the expression for τ given by (9), we conclude that

$$-\frac{\dot{\mathcal{B}}}{\mathcal{B}} = \pi - \delta + \frac{1}{\kappa} \cdot \frac{1}{\pi - \bar{\pi}} \cdot \left[1 - \frac{u}{v(u)} \cdot \frac{1 - u - v(u)}{1 - 2u}\right], \tag{23}$$

where u is the unemployment rate and $v(u)$ is the recruiting rate, given by (6).

3.6. Aggregate demand: Euler equation

We now derive the aggregate demand from optimal consumption and saving. Combining the first-order conditions (19) and (22), we obtain an Euler equation:

$$\frac{\dot{y}}{y} = (i - \pi + \sigma \cdot y) - \delta. \tag{24}$$

In the Euler equation, $i - \pi$ is the real interest rate, which gives the financial return on saving, while $\sigma \cdot y$ is the marginal rate of substitution between wealth and consumption, which gives the hedonic return on saving. Just as in the New Keynesian model developed by Michaillat and Saez (2021b), the presence of wealth in the utility function produces a discounted Euler equation (McKay, Nakamura, and Steinsson 2017).

In steady state ($\dot{y} = 0$), equation (24) yields the Euler curve:

$$y = \frac{\delta - i + \pi}{\sigma} \tag{25}$$

The Euler curve gives the steady-state amount of output demanded by households when they optimally save over time. The preference over social status and wealth, σ, determines the slope of the curve.

When we solve the model we will focus on inflation π and unemployment u, so we rewrite the Euler equation in terms of the unemployment rate u instead of output y. Since $y = (1 - u)l$, we have $\dot{y} = -\dot{u} \cdot l$ and

$$\frac{\dot{y}}{y} = -\frac{\dot{u}}{1 - u}.$$

Accordingly, the Euler equation (24) becomes

$$\frac{\dot{u}}{1 - u} = \delta - \left[i - \pi + \sigma \cdot (1 - u) \cdot l\right] \tag{26}$$

The Euler curve (25) becomes

$$1 - u = \frac{\delta - i + \pi}{\sigma \cdot l} \tag{27}$$

3.7. Aggregate supply: Phillips equation

Next we derive the aggregate supply from optimal pricing. Combining the first-order conditions (21) and (23), we obtain a Phillips equation linking inflation to unemployment:

$$\dot{\pi} = \delta \cdot (\pi - \bar{\pi}) - \frac{1}{\kappa} \cdot \left[1 - \frac{u}{v(u)} \cdot \frac{1 - u - v(u)}{1 - 2u}\right]. \tag{28}$$

In the Phillips equation, the parameter κ is the price-adjustment cost. The term in square bracket measures the inefficiency of the labor market. When the economy is inefficiently tight, $v > u$ so the term is positive. When the economy is efficient, $v = u$ so the term is zero. When the economy is inefficiently slack, $v < u$ so the term is negative.

In steady state ($\dot{\pi} = 0$), equation (28) yields the Phillips curve:

$$\kappa \cdot \delta \cdot (\pi - \bar{\pi}) = 1 - \frac{u}{v(u)} \cdot \frac{1 - u - v(u)}{1 - 2u} \tag{29}$$

The Phillips curve gives the steady-state inflation chosen by households given the competition they face from other households, and the cost they face in changing prices. The price-adjustment cost, κ, determines the slope of curve.

3.8. Special cases

Before moving forward, let's pause to examine a few special cases. Consider the Euler and Phillips curves in a standard unemployment-inflation (u, π) plane.

Without wealth in the utility ($\sigma = 0$), the Euler curve (25) would be horizontal:

$$\pi = i - \delta. \tag{30}$$

This curve just imposes that the real interest rate equals the discount rate. Then inflation π is determined one-for-one by the nominal interest rate i. This is the Fisher effect: a higher interest rate leads to higher inflation. The curve is degenerate because it does not involve unemployment u. The curve would also be horizontal in an output-inflation plane, since it does not involve output.

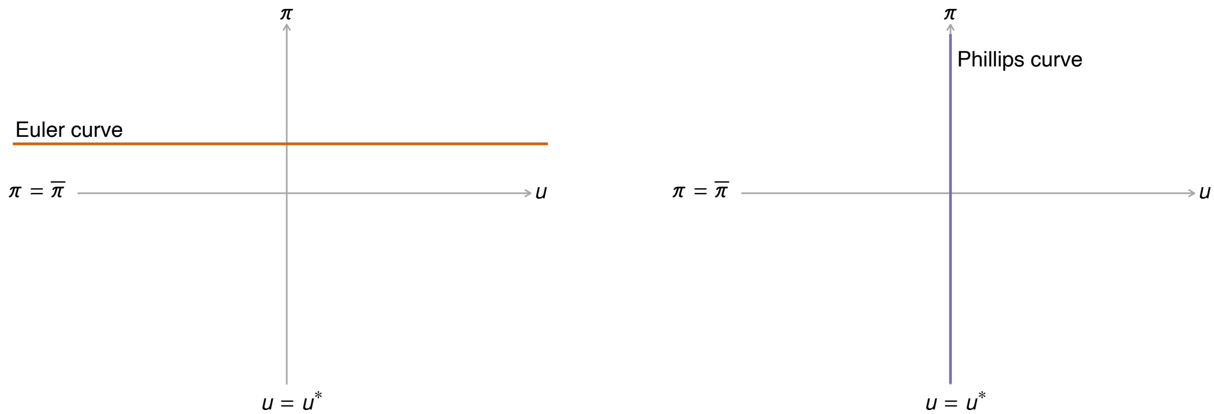
Without price rigidity ($\kappa = 0$), the Phillips curve (29) would be vertical:

$$u = u^*. \tag{31}$$

Indeed, without price rigidity, unemployment and recruiting rates must be equal, so the unemployment rate is efficient ($u = u^*$). The curve would also be vertical in an output-inflation plane, since the unemployment rate pins down the level of output irrespective of inflation: $y = (1 - u^*)l$.

3.9. Divine coincidence

The divine coincidence directly appears in the Phillips equation (29). The equation shows that inflation is on target ($\pi = \bar{\pi}$) if and only if the right-hand side is zero, which happens if and only if unemployment are efficient ($u = v$ so $u = u^*$). Therefore, if the



A. No wealth in the utility

B. No price rigidity

FIGURE 2. Euler and Phillips curves in special cases

A: The Euler curve without wealth in the utility is given by (30). B: The Phillips curve without price rigidity is given by (31).

government is able to bring unemployment to its efficient level, it will also automatically ensure that inflation is on target. In other words, when the government achieves its employment mandate, it also automatically achieves its price mandate.

Both mandates are achieved by moving the Euler curve along the Phillips curve to arrive at the point where $u = u^*$ and $\pi = \bar{\pi}$. This can be done for instance through monetary policy, which affects the nominal interest rate i and therefore the location of the Euler curve (25). The efficient nominal interest rate i^* ensures that inflation is on target ($\pi = \bar{\pi}$) and unemployment is efficient ($u = u^*$). From the Euler curve (25), we obtain an expression for the efficient nominal interest rate i^*:

$$1 - u^* = \frac{\delta - i^* + \bar{\pi}}{\sigma \cdot l}$$

so that the efficient nominal interest rate is

$$i^* = \bar{\pi} + \delta - \sigma \cdot (1 - u^*) \cdot l. \tag{32}$$

When the nominal interest rate is set to i^*, the model admits a steady-state solution in which the divine coincidence prevails: $(\pi, u) = (\bar{\pi}, u^*)$ satisfies both (29) and (25) when i is given by (32).

Such divine steady state exists only when $i^* \geq 0$. If $i^* < 0$, then the divine steady state is not a solution of the model since it would violate the zero lower bound constraint that $i \geq 0$. In that case the central bank would resort to setting $i = 0$.

4. Model dynamics

The model admits a steady-state solution in which the divine coincidence prevails. At that point, inflation is on target and the unemployment rate is efficient. To investigate the dynamics around that steady state, we now linearize the differential equations (26) and (28) around the divine steady state $(\bar{\pi}, u^*)$.

4.1. Linearized model around the divine steady state

We begin by introducing the deviations from efficient steady state: $\hat{u} = u - u^*$ and $\hat{\pi} = \pi - \bar{\pi}$. We also allow the nominal interest rate to follow a Taylor rule:

$$i = i^* + \phi(\pi - \bar{\pi}), \tag{33}$$

where $i^* \geq 0$ is the efficient nominal interest rate and $\phi \geq 0$ is the automatic response of the nominal interest rate to inflation. With the Taylor rule, the Euler equation (26) becomes

$$\frac{\dot{u}}{1-u} = \delta + \phi\bar{\pi} - \left[i^* + (\phi - 1)\pi + \sigma \cdot (1-u) \cdot l\right]$$

Using the value of i^* from (32), we can simplify the Euler equation:

$$\frac{\dot{u}}{1-u} = \sigma \cdot (u - u^*) \cdot l - (\phi - 1)(\pi - \bar{\pi}). \tag{34}$$

We start by linearizing differential equation (34) around $(u^*, \bar{\pi})$. The linearized version is easy to derive since the differential equation is almost linear:

$$\dot{u} = (1 - u^*) \cdot \left[\sigma \cdot l \cdot \hat{u} - (\phi - 1)\hat{\pi}\right]. \tag{35}$$

The linearized version of differential equation (28) around $(u^*, \bar{\pi})$ is a little bit more complicated to derive. The key is to find the partial derivative of

$$\mathcal{P}(u) = -\frac{1}{\kappa} \cdot \left[1 - \frac{u}{\nu(u)} \cdot \frac{1 - u - \nu(u)}{1 - 2u}\right]$$

with respect to u at u^*. This will be the coefficient in front of $\hat{u}$ in the linearized equation. To do that, we need the derivative of

$$\mathcal{Q}(u) = \frac{u}{\nu(u)} \cdot \frac{1 - u - \nu(u)}{1 - 2u}$$

with respect to u at u^*. From (6), we know that the elasticity of $v(u)$ with respect to u is $d\ln(v)/d\ln(u) = -1$, so we have

$$\frac{d\ln(\mathcal{Q})}{d\ln(u)} = 1 + 1 + \frac{-u + v}{1 - u - v} + \frac{2u}{1 - 2u}.$$

When $u = u^*$, we also have $u = v$, so the elasticity simplifies to

$$\frac{d\ln(\mathcal{Q})}{d\ln(u)} = 2\left[1 + \frac{u^*}{1 - 2u^*}\right].$$

Moreover, $Q(u^*) = 1$ so we have the following derivative:

$$\frac{d\mathcal{Q}}{du} = \frac{2}{u^*}\left[1 + \frac{u^*}{1 - 2u^*}\right].$$

Since $\mathcal{P}'(u) = \mathcal{Q}'(u)/\kappa$, we finally get

$$\frac{d\mathcal{Q}}{du} = \frac{2}{\kappa u^*} \cdot \frac{1 - u^*}{1 - 2u^*}.$$

Accordingly, the linearized Phillips curve is

(36) $$\dot{\pi} = \delta\hat{\pi} + \frac{2}{\kappa} \cdot \frac{1 - u^*}{(1 - 2u^*)u^*} \cdot \hat{u}$$

4.2. Classification of the linearized model

The Euler-Phillips system (26)-(28) is nonlinear, but we can determine its properties around the divine steady state from its linearized form. Combining (35) and (36), we find that around the divine steady state $[u, \pi] = [u^*, \bar{\pi}]$, the linearized Euler-Phillips system is

(37) $$\begin{bmatrix} \dot{u}(t) \\ \dot{\pi}(t) \end{bmatrix} = \begin{bmatrix} \sigma y^* & -(\phi - 1)(1 - u^*) \\ 2(1 - u^*)/[\kappa u^*(1 - 2u^*)] & \delta \end{bmatrix} \begin{bmatrix} \hat{u}(t) \\ \hat{\pi}(t) \end{bmatrix}.$$

We denote by $\boldsymbol{M}$ the matrix in (37), and by $\mu_1 \in \mathbb{C}$ and $\mu_2 \in \mathbb{C}$ the two eigenvalues of $\boldsymbol{M}$, assumed to be distinct.

We classify the Euler-Phillips system from the trace and determinant of $\boldsymbol{M}$ (Hirsch, Smale, and Devaney 2013, pp. 61–64). The classification relies on the property that $\mathrm{tr}(\boldsymbol{M}) = \mu_1 + \mu_2$ and $\det(\boldsymbol{M}) = \mu_1\mu_2$. Using (37), we compute the trace and determinant of $\boldsymbol{M}$:

$$\mathrm{tr}(\boldsymbol{M}) = \delta + \sigma y^*$$
$$\det(\boldsymbol{M}) = \delta\sigma y^* + \frac{2(\phi - 1)}{\kappa} \cdot \frac{(1-u^*)^2}{u^*(1-2u^*)}.$$

Clearly, $\mathrm{tr}(\boldsymbol{M}) > \delta > 0$. Further, since $\phi \geq 0$, we have $\phi - 1 \geq -1$, so that

$$\det(\boldsymbol{M}) \geq \delta\sigma y^* - \frac{2}{\kappa}\frac{(1-u^*)^2}{u^*(1-2u^*)}.$$

Just as in Michaillat and Saez (2021b), we assume that the marginal utility of wealth is large enough to ensure that the determinant is positive:

$$\sigma \geq \frac{2}{\kappa\delta l} \cdot \frac{1-u^*}{u^*(1-2u^*)}. \tag{38}$$

Under this assumption, $\mathrm{tr}(\boldsymbol{M}) > 0$ and $\det(\boldsymbol{M}) > 0$, so the Euler-Phillips system is a source for any $\phi \geq 0$. When prices are more flexible (lower κ), the marginal utility of wealth need to be larger to ensure that the determinant is positive and the system is a source.

Indeed $\det(\boldsymbol{M}) > 0$ indicates that μ_1 and μ_2 are either real, nonzero, and of the same sign; or complex conjugates. Since in addition $\mathrm{tr}(\boldsymbol{M}) > 0$, μ_1 and μ_2 must be either real and positive, or complex with a positive real part. Indeed, if μ_1 and μ_2 were real and negative, $\mathrm{tr}(\boldsymbol{M}) = \mu_1 + \mu_2 < 0$. If they were complex with a negative real part, $\mathrm{tr}(\boldsymbol{M}) = \mu_1 + \bar{\mu_1} = 2\,\mathrm{Re}(\mu_1) < 0$.

When μ_1 and μ_2 are real and positive, the solution of the linearized system is

$$[\hat{u}(t), \hat{\pi}(t)] = x_1 e^{\mu_1 t}\boldsymbol{v}_1 + x_2 e^{\mu_2 t}\boldsymbol{v}_2, \tag{39}$$

where $\boldsymbol{v}_1 \in \mathbb{R}^2$ and $\boldsymbol{v}_2 \in \mathbb{R}^2$ are the linearly independent eigenvectors respectively associated with the eigenvalues μ_1 and μ_2, and $x_1 \in \mathbb{R}$ and $x_2 \in \mathbb{R}$ are constants determined by the terminal condition (Hirsch, Smale, and Devaney 2013, p. 35). From (39), we see that the Euler-Phillips system is a source when $\mu_1 > 0$ and $\mu_2 > 0$. The solutions start at 0 when $t \to -\infty$ and go to infinity parallel to $\boldsymbol{v}_2$ when $t \to +\infty$.

When μ_1 and μ_2 are complex conjugates with a positive real part, we write the eigenvalues as $\mu_1 = \mu + i\beta$ and $\mu_2 = \mu - i\beta$ with $\mu > 0$. We also write the eigenvector associated with μ_1 as $\boldsymbol{v}_1 + i\boldsymbol{v}_2$, where the vectors $\boldsymbol{v}_1 \in \mathbb{R}^2$ and $\boldsymbol{v}_2 \in \mathbb{R}^2$ are linearly independent. Then the solution takes a more complicated form:

$$\begin{bmatrix} \hat{u}(t) \\ \hat{\pi}(t) \end{bmatrix} = e^{\mu t}\, [\boldsymbol{v}_1, \boldsymbol{v}_2] \begin{bmatrix} \cos(\beta t) & \sin(\beta t) \\ -\sin(\beta t) & \cos(\beta t) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},$$

where $[\boldsymbol{v}_1, \boldsymbol{v}_2] \in \mathbb{R}^{2\times 2}$ is a 2×2 matrix, and $x_1 \in \mathbb{R}$ and $x_2 \in \mathbb{R}$ are constants determined by the terminal condition (Hirsch, Smale, and Devaney 2013, pp. 44–55). These solutions wind periodically around the steady state, moving away from it as $t \to +\infty$. Hence, the Euler-Phillips system is a spiral source.

Overall, when the marginal utility of wealth is large enough (equation (38)), the linearized model is source whether monetary policy is active ($\phi > 1$) or passive ($0 \geq \phi \geq 1$). This is just as in the New Keynesian model. That model is a source irrespective of monetary policy when the marginal utility of wealth is large enough (Michaillat and Saez 2021b, proposition 1).

4.3. Local uniqueness of the model solution

We assume that the marginal utility of wealth is large enough: equation (38) holds. Therefore, the Euler-Phillips system is a source, which implies that the solution of the model is always locally unique—even when monetary policy is passive. The only solution in the vicinity of the divine steady state is to jump to the steady state and stay there. If the economy jumped somewhere else, unemployment or inflation would diverge away from the steady state.

Unlike in the New Keynesian model, indeterminacy is never a risk, so the central bank does not need to worry about how strongly its policy rate responds to inflation. The central bank can even follow an interest-rate peg without creating indeterminacy.

4.4. Phase diagram

We now construct the phase diagrams of the linearized model to understand its dynamics better.[2] The diagrams are displayed in figure 3.

We begin with the linearized Phillips equation (36), which gives $\dot{\pi}$. First, we plot the

[2]Michaillat and Saez (2021b) show for instance how to use the phases diagrams to study ZLB episodes of finite duration and forward guidance. They also show how to construct sample solutions to the Euler-Phillips system using the phase diagrams.

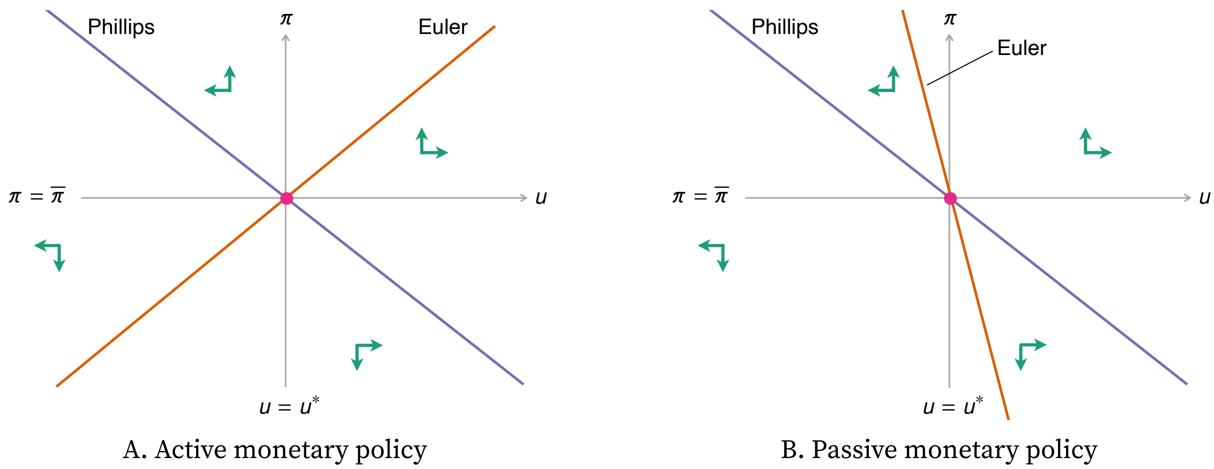


FIGURE 3. Phase diagrams of the linearized model

The figure displays phase diagrams for the dynamical system generated by the linearized Euler equation (35) and Phillips equation (36). The variable u is unemployment; u^* is the efficient rate of unemployment; π is inflation; $\bar{\pi}$ is the inflation target. The Euler curve is the locus $\dot{u} = 0$, given by (41). The Phillips curve is the locus $\dot{\pi} = 0$, given by (40). The monetary-policy rate is given by $i = i^* + \phi(\pi - \bar{\pi})$: when monetary policy is active, $1 < \phi$; when monetary policy is passive, $0 \leq \phi \leq 1$. The figure shows that the linearized model is a source whether monetary policy is active or passive.

locus $\dot{\pi} = 0$, which is the linearized Phillips curve. The locus is given by

$$\hat{\pi} = -\frac{2}{\delta\kappa u^*} \cdot \frac{1 - u^*}{1 - 2u^*} \cdot \hat{u}. \tag{40}$$

The Phillips curve is downward sloping, and goes through the point $[u = u^*, \pi = \bar{\pi}]$. Second, we plot the arrows giving the directions of the trajectories solving the Euler-Phillips system. The sign of $\dot{\pi}$ is given by (36): any point above the Phillips curve (where $\dot{\pi} = 0$) has $\dot{\pi} > 0$, and any point below the curve has $\dot{\pi} < 0$. So inflation is rising above the Phillips curve and falling below it.

We next turn to the linearized Euler equation (35), which gives $\dot{u}$. We plot the locus $\dot{u} = 0$, which we is the linearized Euler curve. The locus is given by

$$\hat{u} = \frac{\phi - 1}{\sigma \cdot l} \cdot \hat{\pi}. \tag{41}$$

The Euler curve goes through the point $[u = u^*, \pi = \bar{\pi}]$. It is downward sloping if $\phi < 1$, vertical if $\phi = 1$, and upward sloping if $\phi > 1$.

Next we use the Euler equation (35) to determine the sign of $\dot{u}$. We first consider an active monetary policy ($\phi > 1$), as showed in figure 3A. Any point above the Euler curve has $\dot{u} < 0$, and any point below it has $\dot{u} > 0$. Hence, in all four quadrants of the phase diagram, the trajectories move away from the steady state. We conclude that the Euler-Phillips system is a source when monetary policy is active.

Second, we consider a passive monetary policy ($\phi \in [0, 1)$), as showed in figure 3B. Now any point above the Euler curve has $\dot{u} > 0$, and any point below it has $\dot{u} < 0$. Nevertheless, in all four quadrants of the phase diagram, the trajectories move away from the steady state. We conclude that the Euler-Phillips system remains a source when monetary policy is passive.

The phase diagrams also illustrate the origin of the condition (38) on the marginal utility of wealth. The Euler-Phillips system remains a source with passive monetary policy as long as the Euler curve is steeper than the Phillips curve in figure 3B. The Euler curve is the most flat with an interest-rate peg ($\phi = 0$), and then its slope is just the marginal utility of wealth. Thus, the marginal utility is required to be above a certain level—which is given by (38).

5. Response to shocks and application to the pandemic

Next, we use the linearized model to study business-cycle shocks. Since the linearized model is a source, the solution of the model is given by the intersection of the linearized Euler curve and linearized Phillips curve. In response to unexpected, permanent shocks, the solution just jumps from the old intersection to the new intersection. Although the model is dynamic, we can therefore study the response to shocks by comparative statics.[3]

5.1. Intuitions behind the Euler and Phillips curves

In this section we use comparative statics to study the effect of macroeconomic shocks in the model. Since we rely on movements of and along the Euler and Phillips curves, it is key to understand what these curves represent—what are the trade-offs they capture.

The Euler curve (41) imposes that the rate of return on wealth equals the time discount rate—otherwise households would not keep their consumption constant. With wealth in the utility function, the returns on wealth are not only financial but also hedonic. As showed by (24), the total rate of return is the real interest rate $r = i - \pi = i_0 + (\phi - 1)\pi$ plus the hedonic rate of return $\sigma y = \sigma(1 - u)l$.

The Euler curve imposes that the real interest rate, which depends on inflation, plus the hedonic rate of return on wealth, which depends on unemployment, equal the time

[3] Of course, the same is true in the New Keynesian model (Michaillat and Saez 2021b, section 5).

discount rate. It thus links inflation to unemployment. When the real interest rate is higher, people have a financial incentive to save more and postpone consumption. They keep consumption constant only if the hedonic returns on wealth fall enough to offset the increase in financial returns: this requires the unemployment rate to rise. As a result, the Euler curve describes unemployment as an increasing function of the real interest rate. When monetary policy is active, the real rate is an increasing function of inflation. Then, the Euler curve describes unemployment as an increasing function of inflation. On the other hand, when monetary policy is passive, the real rate is a decreasing function of inflation. Then, the Euler curve describes unemployment as a decreasing function of inflation.

The Phillips curve (40) explains why inflation is above target whenever the unemployment rate is inefficiently low, and below target whenever the unemployment rate is inefficiently high. When inflation is above target, a seller can reduce its price-adjustment cost by lowering its rate of inflation. Since pricing is optimal, however, there cannot exist any profitable deviation from the current situation. This means that the seller must incur a commensurate cost when it lowers its rate of inflation. With lower inflation, the price charged by the seller drops relative to the prices of other sellers. The absence of profitable deviation imposes that the price reduction must be costly, so the price must already be below the profit-maximizing price. And since local tightness is inversely related to the seller's price, local tightness must be above the profit-maximizing tightness, which is just the efficient tightness of 1 (section 2.10). In other words, the Phillips curve says that when inflation is above target, tightness must be inefficiently high and the unemployment rate must be inefficiently low.

The same logic holds when inflation is below target. Then a seller can reduce its price-adjustment cost by raising its rate of inflation. With higher inflation, the price charged by the seller rises relative to the prices of other sellers. The absence of profitable deviation imposes that the price increase must be costly, so the price must already be above the profit-maximizing price. And since local tightness is inversely related to the seller's price, local tightness must be below the profit-maximizing tightness, which is the efficient tightness of 1. So the Phillips curve says that when inflation is below target, tightness must be inefficiently low and the unemployment rate must be inefficiently high.

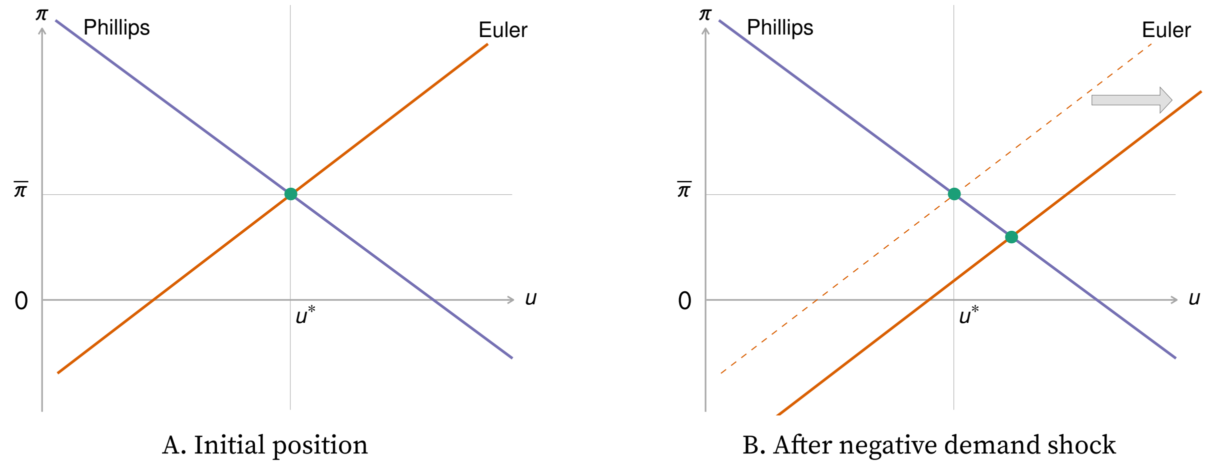


FIGURE 4. Response of the linearized model to a negative demand shock

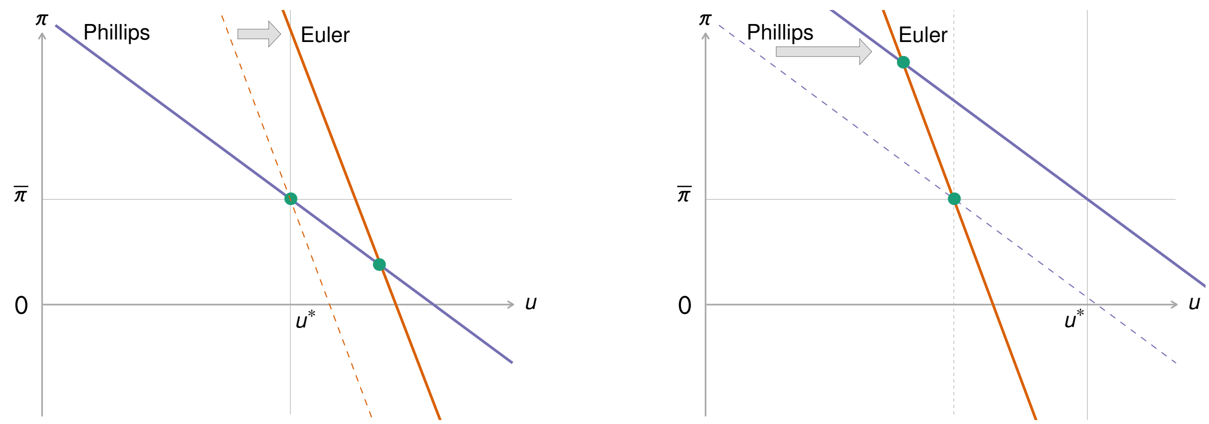
A: The Phillips curve is given by equation (40). The Euler curve is given by equation (41). The variable u is unemployment; u^* is the efficient rate of unemployment; π is inflation; $\bar{\pi}$ is the inflation target. The intersection of the Phillips and Euler curves gives the solution of the model when monetary policy is given by (33) with $\phi > 1$. B: The shock is an unexpected permanent decrease in the discount rate (δ), or an unexpected permanent increase in the marginal utility of wealth (σ), or an unexpected permanent increase in the nominal interest rate (i). The graph shows that unemployment increases while inflation decreases after the shock.

5.2. Typical recession: negative demand shock

We consider first a traditional business-cycle shock: an aggregate-demand shock, which shifts the Euler curve. Such shock could be caused by a change in sentiment, reflected in a different marginal utility of wealth σ or different discount rate δ. A high σ for example indicates a low desire for consumption and therefore produces a low aggregate demand. The shock could also be a change in monetary policy, affecting the nominal interest rate i. A high i makes it more appealing to save and therefore dampens aggregate demand.

We begin by looking at the effect of a negative aggregate-demand shock under the standard assumption that monetary policy is active. In the linearized model, the negative demand shock leads to an outward shift of the Euler curve (figure 4). In response to the negative shock, unemployment is higher, the unemployment gap is also higher, and inflation is lower. The economy is moving along the Phillips curve so there is a negative correlation between unemployment and inflation.

If monetary policy is passive instead of active, the effects of the negative aggregate-demand shock remain the same (figure 5A). The negative demand shock leads to an outward shift of the Euler curve, which raises unemployment and the unemployment gap, and lowers inflation.



A. Negative aggregate-demand shock

B. Negative aggregate-supply shock

FIGURE 5. Response of the linearized model to shocks under passive monetary policy

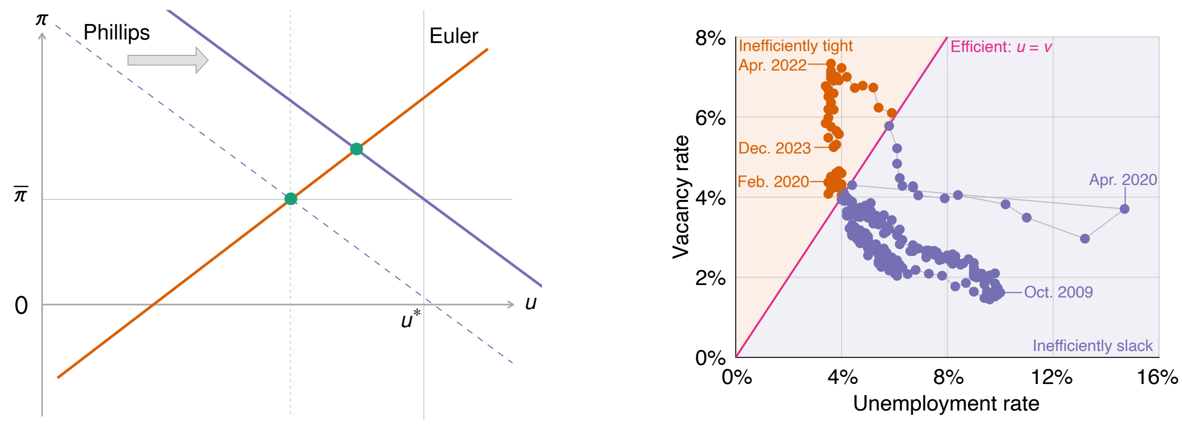
A: The panel reproduces figure 4B when monetary policy is given by (33) with $\phi \in [0,1]$ instead of $\phi > 1$. The responses of unemployment and inflation to the negative aggregate-demand shock remain the same. B: The panel reproduces figure 6A when monetary policy is given by (33) with $\phi \in [0,1]$ instead of $\phi > 1$. The response of inflation to the negative aggregate-supply shock remains the same, but the response of unemployment changes. Unemployment falls after the shock instead of rising.

5.3. Pandemic: negative supply shock

We consider next an unusual business-cycle shock: an aggregate-supply shock, which shifts the Phillips curve. Such shock is caused by a shift in the Beveridge curve (6), so either a change in the job-separation rate s or the matching efficacy ω. Both an increase in separation or a decrease in efficacy shift the Beveridge curve outward, which leads to an increase in the efficient unemployment rate u^*.

We begin by looking at the effect of a negative aggregate-supply shock under the standard assumption that monetary policy is active. In the linearized model, a negative supply shock leads to an outward shift of the Phillips curve (figure 6). In response to the negative shock, unemployment is higher, and inflation is higher. But the key is that the unemployment gap is lower (it has become negative) and inflation is higher. Indeed the efficient unemployment rate has increased more than actual unemployment, so the unemployment rate is now inefficiently low. Such excessive tightness leads to higher inflation.

If monetary policy is passive instead of active, the effects of the negative aggregate-supply shock are mostly but not entirely the same (figure 5A). The negative supply shock leads to an outward shift of the Euler curve, which raises inflation. Unlike with active monetary policy, here the unemployment rate actually falls after the shift of the Euler curve. Since the unemployment rate decreases but the efficient unemployment



A. After negative supply shock

B. Pandemic shift of the US Beveridge curve

FIGURE 6. Response of the linearized model to a negative supply shock

A: The Euler and Phillips curves are constructed in figure 4A. The shock is an unexpected permanent decrease in the matching efficacy (ω), or an unexpected permanent increase in the job-separation rate (s). The graph shows that both unemployment and inflation increase after the shock. B: The Beveridge curve was produced by Michaillat and Saez (2023, figure 8).

rate increases, the unemployment gap unambiguously increases. Just like with active monetary policy, the flare-up in inflation is associated with an inefficiently tight labor market.

If the Euler curve remains the same after the outward shift of the Phillips curve—for instance because the central bank is not aware of it—then we obtain a burst of inflation after the adverse shock to the Phillips curve (figure 6A). Since the US Beveridge curve has shifted dramatically outward in the aftermath of the coronavirus pandemic (figure 6B), the flare-up in inflation in 2021–2023 might partly result from this dramatic shift.

6. Adding a kink to the Phillips curve

The evidence provided by Benigno and Eggertsson (2023) suggests not only that the divine coincidence holds in the United States, but also that the Phillips curve has a kink at the point of divine coincidence (figure 1A). The Phillips curve appears steeper when the labor market is inefficiently tight and flatter when the labor market is inefficiently slack. This section shows how a kink can be added to the Phillips curve. In the model, the kink appears because wage cuts are more painful to workers than price increases are to consumers.

6.1. Asymmetric price-adjustment cost

So far we have assumed that the price-adjustment cost is symmetric, just as in the original work by Rotemberg (1982). It is not unreasonable to assume that the price-adjustment cost is asymmetric, however, because in the model the cost of a price increase and the cost of a price decrease capture different phenomena, affecting different people.

When prices fall, or increase less than normal, workers in household k feel shortchanged. Indeed, the price p_k is their hourly salary. And Bewley (1999, 2005) has shown that workers' morale dips when their wage does not grow as expected. So here we assume that workers incur a quadratic cost when wage growth $\pi_k(t)$ falls short of the normal growth $\bar{\pi}$.

When prices rise, or increase more than normal, it is the customers of household k that are unhappy. Shiller (1996) shows that higher-than-normal inflation upsets customers, who feel unfairly treated when they go to the store. In fact, such inflation makes customers angry at the sellers. So here we assume that sellers internalize the anger of customers that is directed at them, and incur another quadratic cost when wage growth $\pi_k(t)$ is above the normal growth $\bar{\pi}$.[4]

Formally, if $\pi_k > \bar{\pi}$, the flow disutility caused by prices deviating from the norm is

$$\rho(\pi_k) = \frac{\kappa^+}{2} \cdot [\pi_k - \bar{\pi}]^2.$$

This cost reflects the fact that higher-than-normal prices upset customers. If $\pi_k < \bar{\pi}$, the flow disutility caused by prices deviating from the norm is

$$\rho(\pi_k) = \frac{\kappa^-}{2} \cdot [\pi_k - \bar{\pi}]^2.$$

This cost reflects the fact that lower-than-normal wages damage workers' morale.

The parameters $\kappa^+ > 0$, $\kappa^- > 0$ govern the price-adjustment costs. Since the costs come from different sources when inflation is too high and too low, we allow the cost parameters κ^+ and κ^- to be different. We postulate that workers' anger at wage cuts is stronger than customers' anger at price increases, so we assume

$$\kappa^- > \kappa^+. \tag{42}$$

[4]This is a reduced-form way to capture how sellers internalize customers' anger at price increases. For a complete model of why price increases anger customers, and how firms internalize such anger to maximize profits, see Eyster, Madarasz, and Michaillat (2021).

This gap between κ^+ and κ^- will generate a kink in the Phillips curve.[5]

Even with $\kappa^- > \kappa^+$, the cost function $\rho(\pi)$ is continuous and differentiable at $\bar{\pi}$ since

$$\lim_{\pi \to \bar{\pi}^+} \rho(\pi) = \lim_{\pi \to \bar{\pi}^-} \rho(\pi) = 0$$
$$\lim_{\pi \to \bar{\pi}^+} \rho'(\pi) = \lim_{\pi \to \bar{\pi}^-} \rho'(\pi) = 0,$$

so that we can complete the definition of the cost function at $\bar{\pi}$ by $\rho(\bar{\pi}) = 0$ and $\rho'(\bar{\pi}) = 0$.

6.2. Kink in the Phillips curve

Although we assume that the price-adjustment cost is asymmetric, all the derivations remain the same. As a result, the Phillips equation (28) is now defined piecewise. For $\pi < \bar{\pi}$, the Phillips equation is

$$\dot{\pi} = \delta \cdot (\pi - \bar{\pi}) - \frac{1}{\kappa^-} \cdot \left[1 - \frac{u}{v(u)} \cdot \frac{1 - u - v(u)}{1 - 2u}\right]. \tag{43}$$

And for $\pi > \bar{\pi}$, the Phillips equation is

$$\dot{\pi} = \delta \cdot (\pi - \bar{\pi}) - \frac{1}{\kappa^+} \cdot \left[1 - \frac{u}{v(u)} \cdot \frac{1 - u - v(u)}{1 - 2u}\right]. \tag{44}$$

Accordingly, the linearized Phillips equation (36) is also defined piecewise. For $\pi < \bar{\pi}$, the linearized Phillips equation is

$$\dot{\pi} = \delta\hat{\pi} + \frac{2}{\kappa^-} \cdot \frac{1 - u^*}{(1 - 2u^*)u^*} \cdot \hat{u}. \tag{45}$$

And for $\pi > \bar{\pi}$, the linearized Phillips equation is

$$\dot{\pi} = \delta\hat{\pi} + \frac{2}{\kappa^+} \cdot \frac{1 - u^*}{(1 - 2u^*)u^*} \cdot \hat{u}. \tag{46}$$

Because of the shape of the linearized Phillips equation, the linearized Phillips curve, obtained by setting $\dot{\pi} = 0$ in (45) and (46), has a kink at the point $\pi = \bar{\pi}$ and $u = u^*$. For

[5] The assumption of quadratic price-adjustment costs is extremely popular in the New Keynesian literature, but a few papers use asymmetric price-adjustment costs—albeit with a different functional form than here. For instance, Cao, Luo, and Nie (2023, equation 14) assume an asymmetric price-adjustment cost to remove episodes of extreme deflation from their model. Cao, Diba, and Lee (2023) use the same asymmetric price-adjustment cost to resolve the anomalies of the New Keynesian model at the zero lower bound.

$\hat{\pi} < 0$ and $\hat{u} > 0$, the linearized Phillips curve is

(47) $$\hat{\pi} = -\frac{2}{\delta\kappa^{-}} \cdot \frac{1-u^{*}}{(1-2u^{*})u^{*}} \cdot \hat{u}.$$

And for $\hat{\pi} > 0$ and $\hat{u} < 0$, the linearized Phillips curve is

(48) $$\hat{\pi} = -\frac{2}{\delta\kappa^{+}} \cdot \frac{1-u^{*}}{(1-2u^{*})u^{*}} \cdot \hat{u}.$$

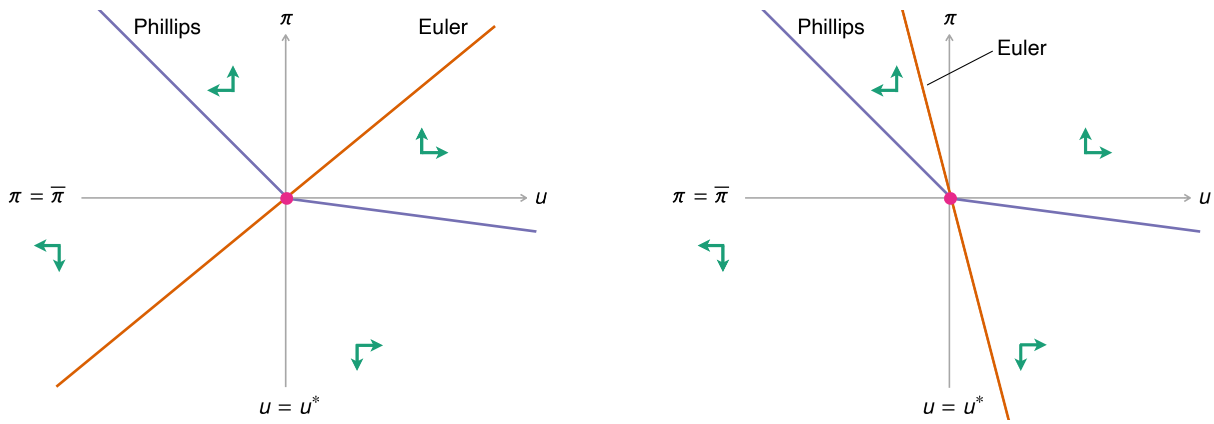
Since $\kappa^{-} > \kappa^{+}$, the branch (48) of the linearized Phillips curve is steeper than the branch (47). The linearized Phillips curve is therefore downward sloping with a kink at the point $[u = u^{*}, \pi = \bar{\pi}]$.

Because $\kappa^{-} > \kappa^{+}$, the Phillips curve is steeper when $u < u^{*}$ and flatter when $u > u^{*}$. Such kink has been observed in aggregate US data (Benigno and Eggertsson 2023), as showed in figure 1A, but also in metropolitan US data (Babb and Detmeister 2017; Smith, Timmermann, and Wright 2023; Gitti 2023), and in international data (Smith, Timmermann, and Wright 2023; Benigno and Eggertsson 2024).

Previous papers have proposed models of a kinked Phillips curve. Benigno and Eggertsson (2023) and Gitti (2023) develop New Keynesian models in which the kink arises from differential wage rigidity: wages are rigid when tightness is below 1 but flexible when tightness is above 1. In an old-fashioned Keynesian model, Chiarella et al. (2003) obtain a kink from a related assumption: that nominal wages are rigid downward. Here the kink comes from a different assumption: that wage decreases are more costly to producers than price increases.

The novelty of our model is that it guarantees that the kink occurs at the efficient unemployment rate—it guarantees that the divine coincidence holds. This is directly consistent with the findings by Benigno and Eggertsson (2023). But it is also in line with the findings by Babb and Detmeister (2017, Table 4) and Smith, Timmermann, and Wright (2023, Table 5), who find a kink in the US Phillips curve at an unemployment rate of 4.2%. Indeed, in the postwar period in the United States, the efficient unemployment rate is just 4.2% (Michaillat and Saez 2023, section 3.1). So the kink identified by Babb and Detmeister (2017) and Smith, Timmermann, and Wright (2023) occurs right at the efficient unemployment rate.

Despite the kink, the phase diagram of the linearized model retain the same properties (figure 7). Indeed, even with the kink, the arrows giving the directions of the trajectories solving the Euler-Phillips system are the same. The sign of $\dot{\pi}$ is given by (46)



A. Active monetary policy

B. Passive monetary policy

FIGURE 7. Phase diagrams of the linearized model with a kinked Phillips curve

This figure reproduces figure 3 when the Phillips equation has a kink. Instead of being given by (36), the linearized Phillips equation is given by (45) when $\pi < \bar{\pi}$ and by (46) when $\pi > \bar{\pi}$. Even with a kinked Phillips curve, the linearized model is a source whether monetary policy is active or passive.

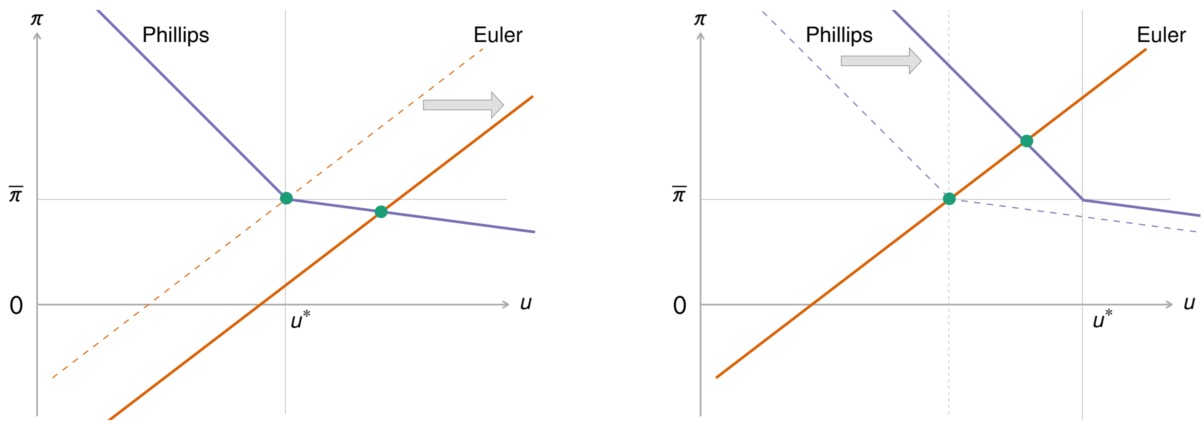
and (45). It remains the case that any point above the Phillips curve (where $\dot{\pi} = 0$) has $\dot{\pi} > 0$, and any point below the curve has $\dot{\pi} < 0$. So inflation is rising above the Phillips curve and falling below it, even with the kink. As a result, the linearized model remains a source, whether monetary policy is active (figure 7A) or passive (figure 7B).

6.3. Implications of the kink

Since the linearized model remains a source with the kink, the solution of the linearized model remains given by the intersection of the linearized Euler curve and linearized Phillips curve. In response to unexpected, permanent shocks, the solution just jumps from the old intersection to the new intersection. We can therefore continue to study the response to shocks by comparative statics.

While the kink in the Phillips curve does not change the comparative statics qualitatively, it does have quantitative consequences. It implies that, starting from a divine situation, a negative aggregate-demand shock will only have a small negative effect on inflation. This is because the economy is moving along the flat branch of the Phillips curve (figure 8A. By contrast, a negative aggregate-supply shock will have a large positive effect on inflation. This is because then the economy is moving along the steep branch of the Phillips curve (figure 8B.

More generally, when the economy is inefficiently tight ($\hat{u} < 0$), any shock tends to generate larger movements in inflation and smaller movements in unemployment.



A. Negative aggregate-demand shock B. Negative aggregate-supply shock

FIGURE 8. Response of the linearized model to shocks with a kinked Phillips curve

This figure reproduces figure 4B and figure 6A when the Phillips equation has a kink. Instead of being given by (40), the linearized Phillips curve is given by (47) when $\pi < \bar{\pi}$ and by (48) when $\pi > \bar{\pi}$. Despite the kinked Phillips curve, unemployment and inflation have the same qualitative responses to shocks.

when the economy is inefficiently slack ($\hat{u} > 0$), any shock tends to generate smaller movements in inflation and larger movements in unemployment.

7. Conclusion

Summary. The divine coincidence has recently appeared in US data. That is, inflation appears on target whenever the labor market is efficient (figure 1). To explain this phenomenon, we propose a model of the divine coincidence. The key is to combine Moen (1997)'s directed search with Rotemberg (1982)'s price-adjustment costs. These assumptions generate a nonvertical Phillips curve that goes through the point of divine coincidence—where unemployment is efficient and inflation is on target.

Positive implications. The model predicts that inflation rises above target whenever the labor market is inefficiently tight. The model therefore provides an explanation for the flare-up in inflation in 2021–2023. Indeed, the US labor market has been inefficiently tight in the aftermath of the coronavirus pandemic—in fact tighter than at any point since the end of World War 2 (Michaillat and Saez 2023). Such excessive tightness must have fueled the flare-up in inflation, especially if the Phillips curve is kinked: steeper when the economy is inefficiently tight and flatter when it is inefficiently slack. Such a kink appears in US data (Babb and Detmeister 2017; Smith, Timmermann, and Wright

2023; Benigno and Eggertsson 2023). The model produces a kinked Phillips curve when we assume that wage cuts are more costly to producers than price hikes.

Normative implications. The divine coincidence matters a great deal for policy because it implies that the full-employment and price-stability mandates of the Federal Reserve coincide. According to the model, if the Fed is able to maintain the economy at full employment, it will also maintain inflation on target, thus satisfying its dual mandate. And to maintain the economy at full employment, the Fed can rely on the sufficient-statistic formula developed by Michaillat and Saez (2022, equation (31)). From any initial nominal interest rate and unemployment gap, the formula gives the change in nominal interest rate required to bring the unemployment gap to zero.

Choice of target variables. In the model, since the full-employment and price-stability mandates of the Fed coincide, it is completely equivalent to aim for the inflation target or the efficient unemployment rate. In practice, however, the Fed should target the variable that is the most volatile, so as to observe more clearly departures from the dual mandate. The slope of the Phillips curve in turn determines which of inflation and unemployment is the most volatile. If the Phillips curve is steep, aggregate-demand shocks will mostly generate movements in inflation, and targeting inflation will be easier. By contrast, if the Phillips curve is flat, aggregate-demand shocks will mostly generate movements in unemployment, and targeting unemployment will be easier. In practice, it seems that the Phillips curve is kinked (figure 1): flat when unemployment is inefficiently high, and steep when unemployment is inefficiently low. The implication is that the Fed should target the efficient unemployment rate u^* when the economy is inefficiently slack; and it should target an inflation rate of 2% when the economy is inefficiently hot.

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Appendix A. Expression for the upper bound on tightness

This appendix expresses the upper bound on tightness, $\bar{\theta}$, as a function of the lower bound on tightness, $\underline{\theta}$. The upper bound on tightness is implicitly defined by

$$q(\bar{\theta}) = s,$$

where $q(\theta)$ is the worker-finding rate and s is the job-separation rate.

Indeed, using (4), we write $q(\bar{\theta}) = s$ as

$$\frac{\omega}{\sqrt{\theta}} - \frac{s}{\theta} - s = 0.$$

With a change of variable $x = 1/\sqrt{\theta}$, this is equivalent to solving the second-order polynomial equation

$$-sx^2 + \omega x - s = 0.$$

The determinant of the equation is $\Delta = \omega^2 - 4s^2 > 0$. The two solutions of the equation are

$$x' = \frac{\omega \pm \sqrt{\Delta}}{2s} = \frac{\omega}{2s} \cdot \left[1 \pm \sqrt{1 - \frac{4s^2}{\omega^2}}\right] = \frac{1 \pm \sqrt{1 - \underline{\theta}}}{\sqrt{\underline{\theta}}}.$$

Accordingly, the tightnesses that solve $q(\theta) = s$ are given by $1/(x')^2$ or

$$\theta' = \frac{\underline{\theta}}{\left[1 \pm \sqrt{1 - \underline{\theta}}\right]^2}$$

The only solution that is larger than $\underline{\theta}$ is

$$\bar{\theta} = \frac{\underline{\theta}}{\left[1 - \sqrt{1 - \underline{\theta}}\right]^2}.$$